The Mathematics Behind the Coronavirus Spread

As of January 31, 2020, the Coronavirus is spiraling out of control in China and is having deep impacts to humanity in that country and possibly the rest of the world. The economic implications are equally as frightening. Let us examine what is occurring mathematically with the spread of the virus and while this is a blog for calculus teachers and students, I will attempt to keep it away from that vein as long as I can.

We have all heard the term exponential growth. In the media, the term is frequently used incorrectly. Many websites report that the virus is growing exponentially. What does that really mean and is it true? Are there other types of growth that are similar? Let’s explore an example of exponential growth. Growing up, we have all heard some variant of this story:

There was once a king in India who was a big chess enthusiast and had the habit of challenging visitors to a game of chess. One day a traveling sage was challenged by the king. The sage having played this game all his life all the time with people all over the world gladly accepted the King’s challenge. To motivate his opponent the king offered any reward that the sage could name. The sage modestly asked just for a few grains of rice in the following manner: the king was to put a single grain of rice on the first chess square and double it on every consequent one. The king accepted the sage’s request.

Having lost the game and being a man of his word the king ordered a bag of rice to be brought to the chess board. Then he started placing rice grains according to the arrangement: 1 grain on the first square, 2 on the second, 4 on the third, 8 on the fourth and so on.

Following the exponential growth of the rice payment, the king quickly realized that he was unable to fulfill his promise because on the twentieth square the king would have had to put 1 million grains of rice. On the 40th square, the king would have had to put 1 billion grains of rice. And, finally, on the 64th square, the king would have had to put more than 18,000,000,000,000,000 grains of rice which is equal to about 210 billion tons and is allegedly sufficient to cover the whole territory of India with a meter-thick layer of rice.

It was at that point that the sage told the king that he doesn’t have to pay the debt immediately but can do so over time. And so the sage became the wealthiest person in the world.

The story uses the expression exponential growth. What does that mean? It is based on a concept taught in precalculus courses and in greater depth in AP calculus: differential equations. Something growing exponentially means that its change over time is proportional to the amount of it that is present now. That means that the more there is, the faster it grows.

In the case of the chess problem, we start with one grain of rice on the first square and the next square contains 2. Thus it has changed by 1 grain. The next square contains 4, so its change is 2. The next square contains 8 so its change is 4. So the more grains that are on any square, the greater the change for the next one. And this growth theoretically continues forever. In this case, it will continue for the 64 squares.

In calculus, this situation can be written as a differential equation where \( P \) represents a population (in this case number of grains of rice), \( t \) represents time, and \( k \) represents a constant: \[
\frac{dP}{dt} = kP. \quad \frac{dP}{dt}
\] represents the change of \( P \) over time.
However, it is important to realize that in this situation, there really is no time factor. We are interested in the change in the number of grains of rice as we move from square to square. There is no fractional value for the square number. So, assuming that $s$ represents the square number and $R$ represents the number of grains of rice on that square, and $k = 1$, we could change the differential equation to $\frac{dR}{ds} = 1R = R$. Below is a table to see how these values change as we move from square to square.

<table>
<thead>
<tr>
<th>$s$ (square)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$ (grains of rice)</td>
<td>1</td>
<td>1+1=2</td>
<td>2+2=4</td>
<td>4+4=8</td>
<td>8+8=16</td>
<td>16+16=32</td>
</tr>
<tr>
<td>$\frac{dR}{ds}$ (change in $R$ for next square)</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>16</td>
<td>32</td>
</tr>
</tbody>
</table>

The number of grains of rice $R$ on each square in the table is doubling for each additional square. How many would be on the 64th square? We don’t want to extend the table out that far but hopefully you can see a pattern. The number of grains of rice on each square is given by the equation $R = 2^{s-1}$. So when $s = 7$, $R = 2^7-1 = 2^6 = 32$. When $s = 8$, $R = 2^8-1 = 2^7 = 64$. Using this formula, when $s = 64$, $R = 2^{64-1} = 2^{63} = 9.22 \times 10^{18}$.

Also remember that this represents the number of grains of rice on each square, not the total number of grains of rice on the entire board. We add one row to the table to get $T$, the total number of grains of rice on the board for any value of $s$.

<table>
<thead>
<tr>
<th>$s$ (square)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
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<td>1</td>
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<td>4+4=8</td>
<td>8+8=16</td>
<td>16+16=32</td>
</tr>
<tr>
<td>$\frac{dR}{ds}$ (change in $R$ for next square)</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>16</td>
<td>32</td>
</tr>
<tr>
<td>$T$ (total number of grains on board)</td>
<td>1</td>
<td>1+2=3</td>
<td>3+4=7</td>
<td>7+8=15</td>
<td>15+16=31</td>
<td>31+32=63</td>
</tr>
</tbody>
</table>

To find the total number of grains of rice on the board after 64, we don’t want to extend the table, but again, we see that there is a pattern. The number of grains of rice on each the entire board is given by the equation $T = 2^s - 1$. So when $s = 7$, $T = 2^7 - 1 = 128 - 1 = 127$. When $s = 8$, $T = 2^8 - 1 = 256 - 1 = 257$. So when $s = 64$, $T = 2^{64} - 1 \approx 1.84 \times 10^{19}$.

This is an example of **discrete exponential growth**. We use the word discrete to indicate that there are no fractional values of $s$. Graphically, these are simply data points. We may connect these points with straight lines to help us view the growth as shown to the right, but there is no value of $T$ at any values of $s$ that are not whole numbers.
Exponential growth will be modeled by an equation in the form \( y = a^x \), where \( a > 1 \). The larger the value of \( a \), as shown in the figure to the right, the faster the graph “explodes.”

It is important to realize that the number of grains of rice on the board at \( s = 63 \), \( T \approx 1.84 \times 10^{19} \) is amazingly large. Allowing one second to place each grain of rice on the board, it would take over 5 trillion centuries (5,849,424,174 to be exact) to place all the grains of rice on the board. That represents well more than the number of grains of rice that ever existed on the earth. In fact, it represents well more than the number of grains of sand that ever existed on the earth.

So the number of grains of rice owned by the king must be limited. If the king owned 1 million grains of rice, then it can be shown that the king will run out of rice after the 19\(^{th}\) square is filled. If the king owned 1 billion grains of rice, then it can be shown that the king will run out of rice after the 29\(^{th}\) square is filled.

**A different type of growth:** Suppose the king has 1 million grains of rice and the agreement now is that the change in the number of grains of rice that will be placed on any square is equal to \( \frac{1}{500000} \) of the product of the number of grains on the board and how many grains that are still available.

For example: We place one grain of rice on the first square. Since there are a million grains of rice in total, there are 999,999 grains available. We find \( \frac{1}{500000} \times 999999 = 2 \). That represents the change in the number of grains of rice as we switch to the 2\(^{nd}\) square and thus, there will be \( 1 + 2 = 3 \) grains of rice on the board after 2 squares.

For the next change, we have used up 3 grains so there are 999,997 grains available. We find the value of \( \frac{3(999997)}{500000} \approx 6 \). That represents the change in the number of grains of rice as we switch to the 3\(^{rd}\) square and thus, there will be \( 3 + 6 = 9 \) grains of rice on the board after 3 squares.

For the next change, we have used up 9 grains so there are 999,991 grains available. We find the value of \( \frac{9(999991)}{500000} \approx 18 \). That represents the change in the number of grains of rice as we switch to the 4\(^{th}\) square and thus, there will be \( 9 + 18 = 27 \) grains of rice on the board after 4 squares.

Below is a table that indicates the total number of grains of rice on the board after \( s \) squares have been filled.

<table>
<thead>
<tr>
<th>( s ) (square)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
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<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R ) (total grains)</td>
<td>1</td>
<td>3</td>
<td>9</td>
<td>27</td>
<td>81</td>
<td>243</td>
<td>729</td>
<td>2,185</td>
<td>6,547</td>
<td>19,554</td>
<td>57,898</td>
<td>166,989</td>
<td>445,199</td>
<td>939,192</td>
</tr>
</tbody>
</table>

A sharp-eyed student will pick up that this data seems to be tripling from square to square. It appears then to adhere perfectly to the equation \( R = 3^{s-1} \) and it does, at least for a while.
So the data appears to be exponential until \( s = 7 \) and then starts to deviate slightly, and then goes completely off the rails. What is going on?

Graphing the data points, we see the pattern of both, the real points in blue and the exponential curve \( R = 3^{s-1} \) in red. They match almost perfectly (based on scale) through \( s = 12 \) but they begin to deviate. Being exponential, the red curve will continue upwards getting steeper and steeper forever, but the blue curve seems to be beginning to level off.

When we usually talk about exponential growth, its equation is usually written as a function of time \( t \):

\[
y = a^t, a > 1.
\]

It is important to realize that in very few situations, exponential growth will continue forever. For instance, people might start moving into a small town and the growth might appear exponential. However, as the town gets more and more crowded, the small town becomes a big town and people become uncomfortable. So the exponential growth slows and perhaps even stops.

If you drop a piece of bread on your kitchen floor, you might see a huge number of ants swarming on top of it and the number of ants might grow exponentially. But as they consume the bread, the food supply is gone, and the exponential growth stops.

You may take a speed-reading course and the speed in which you read a chapter of a book might increase exponentially. But you can only become so fast as a reader and eventually the rapid growth will slow down as you reach your maximum target reading speed.

A new popular product like the Apple Watch comes out and people can’t wait to buy it. The growth of the product is extremely fast. But that growth will not continue forever. Eventually the growth will level off as more and more people own it and new products hit the market.

Whether we are talking about people in a town, ants multiplying, your skill at an activity, or the popularity of a new product, the numbers will not increase without bound. It is possible that growth can start out exponentially, but eventually, it is going to level off. We call this **logistic growth**.

Logistic growth, usually expressed as a function of time, has the differential equation \( \frac{dP}{dt} = kP(C - P) \) where \( k \) is a constant, \( P \) represents a population, \( C \) represents the **Carrying Capacity** (the maximum value of \( P \)), and \( \frac{dP}{dt} \) represents the change in a population. In the example above, since the king had only 1 million grains of rice, \( C = 1,000,000 \). In the case above, our differential equation (DEQ) would be

\[
\frac{dR}{dt} = \frac{1}{500000} R(1000000 - R).
\]
The solution to this DEQ is a complicated expression:

\[ R = \frac{1000000e^{2x}}{e^{2x} + 7.39(1000000)} \]

We care less about the formula than the shape of this logistic curve, shown in purple. It has a distinctive S-shaped curve to it. The growth is close to exponential at first and grows very steeply, but as the curve gets higher, the room for growth is less (as there are only 1,000,000 grains of rice available). This is classic logistic growth. There is a big difference between the real data points and the logistic curve because this graph is based on continuous growth as a function of time, and not the discrete squares which can only be whole numbers.

For instance, if someone spread a rumor that the school was closing early due to a storm, there would be very few people (maybe only one) who knew the rumor at first. Most of the people in the school are hearers, not tellers. As a teller spreads the rumor, a hearer becomes a teller. So the tellers increase and of course, the rumor spreads faster. At a certain moment in time, half the people in the school will be hearers and half will be tellers.

There is a finite number of people in the school (its carrying capacity). But then the growth starts to slow because as more and more people know the rumor, there are fewer people who don’t know it. Eventually, it will get to the point where there are 95% tellers and only 5% hearers. At this point the growth is extremely slow. The figure shows how the growth in people knowing the rumor increases but then decreases and levels off just below the carrying capacity.

That is the pattern in logistic growth. The curve is always increasing (growing). The growth at first is slow (meaning that the its slope is close to zero) but then increases quickly. However at a certain point in time (called the inflection point), the growth begins to slow down and by the time the curve approaches the carrying capacity the growth is almost nonexistent (meaning that its slope is close to zero).

So, summing up, we are examining two types of growth. First, we have exponential growth which continues forever, at least theoretically. But because in most situations, there are only so many people who might live in a town, only so many bacteria that can sustain itself, only so popular a new product might become, etc., most situations which start out as exponential growth become the second type of growth: logistic growth.

So with that background established, let’s investigate the Coronavirus.

As of January 31, 2020 at 7 PM, there were a total of 11,374 confirmed cases in Mainland China.

[https://gisanddata.maps.arcgis.com/apps/opsdashboard/index.html#/bda7594740fd40299423467b48e9ecf6](https://gisanddata.maps.arcgis.com/apps/opsdashboard/index.html#/bda7594740fd40299423467b48e9ecf6)

Here is the data as shown by that website (Center for Systems Science and Engineering at Johns Hopkins University): Note this these are the number of confirmed cases of Coronavirus, not deaths from it.

<table>
<thead>
<tr>
<th>Date</th>
<th>Jan 20</th>
<th>Jan 21</th>
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<th>Jan 28</th>
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<tr>
<td>Day</td>
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<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>Cases</td>
<td>278</td>
<td>326</td>
<td>547</td>
<td>639</td>
<td>916</td>
<td>2,000</td>
<td>2,700</td>
<td>4,400</td>
<td>6,000</td>
<td>7,700</td>
<td>9,700</td>
<td>11,374</td>
</tr>
</tbody>
</table>
To the right are these data points graphed: The first thing we should realize is that unlike the rice problem, this is not a discrete growth problem. Time, measured in days, is continuous. It now makes sense to talk about day 1.5 or day 12.8. So we start by drawing lines between the points, although we know nothing about how many cases of Coronavirus there were in Mainland China between these times, based on the data in this website.

The big question is: are we seeing exponential growth or logistic growth?

Exponential growth curves get steeper and steeper over time. As mentioned before, exponential growth rarely continues forever. There is a finite number of people in China and if everyone gets Coronavirus, the growth of the virus in that country will of necessity stop. However, if logistic growth is occurring, most likely a smaller number than China’s population will end up with the virus.

Which is it? There are not a lot of points with which to make our judgement.

Using our calculators, we force an exponential growth model and a logistic growth model to the points. That is, we try and determine an equation describing each model that will best fit our data points.

This is called regression and is beyond the scope of this article. But we give the results:

For exponential growth, we get the equation:
\[
\text{Cases} = 177.187 (1.440^{\text{day}})
\]

For logistic growth, we get the equation:
\[
\text{Cases} = \frac{15152}{1 + 153.59e^{-0.510\cdot\text{day}}}
\]

To the right is the graph of both equations with the exponential in green and the logistic in red.

It should be obvious that based on these few numbers of points, that the logistic model seems to fit much better. Let’s extrapolate (the action of estimating or concluding something by assuming that existing trends will continue, or a current method will remain applicable) and take both models out to 20 days. The exponential model is off the scale while the logistic model seems to be leveling off with just about 15,000 people from China having the Coronavirus.
Let’s look at our data again but let’s add on the growth (the difference in virus cases from day to day).

<table>
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<tr>
<th>Date</th>
<th>Jan 20</th>
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<th>Jan 30</th>
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<tr>
<td>Day</td>
<td>1</td>
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<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
</tr>
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<td>Cases</td>
<td>278</td>
<td>326</td>
<td>547</td>
<td>639</td>
<td>916</td>
<td>2,000</td>
<td>2,700</td>
<td>4,400</td>
<td>6,000</td>
<td>7,700</td>
<td>9,700</td>
<td>11,374</td>
</tr>
<tr>
<td>Growth</td>
<td>48</td>
<td>221</td>
<td>93</td>
<td>277</td>
<td>1,084</td>
<td>700</td>
<td>1,700</td>
<td>1,600</td>
<td>1,700</td>
<td>2,000</td>
<td>1,674</td>
<td></td>
</tr>
</tbody>
</table>

It appears that for the most part, the average growth is increasing through January 30, although it is about the same from January 27 – January 29. It is only from January 30 – 31 that we see a slowing down of the growth rate (it is still high but not quite as high). It is the appearance of the January 31 data point that is pushing the model towards the logistic growth rate with carrying capacity of about 15,000.

It is far too soon to make predictions. Extrapolation is dangerous with such few data points. But based on this data, it appears that the quarantine might be having some effect and hopefully the exponential growth model is being staved off.

15,000 cases are nothing to sneeze at (pardon the pun). However, it is a far cry from what could happen. To eliminate or reduce growth, it has to be stopped before it gets a foothold. It does not appear that this happened in China. This is an article that appeared in the Sunday February 2 NY times.

https://www.nytimes.com/2020/02/01/world/asia/china-coronavirus.html?nl=todaysheadlines&emc=edit_th_200202&campaign_id=2&instance_id=15505&segment_id=20913&user_id=cc440f4420a4c9095747cc8024199718&regi_id=414578950202

Let me point out again that this is based on the numbers from the Johns Hopkins Center for Systems Science and Engineering. They get their data from the World Health Organization.

https://www.who.int/emergencies/diseases/novel-coronavirus-2019

It is hard to say how accurate it is. The Center for Disease Control and Prevention https://www.cdc.gov/coronavirus/2019-ncov/index.html has not been permitted into China. There are areas within Mainland China that are quite remote, and it is not likely that any kind of testing has been done there. So the actual figures of coronavirus cases may be higher, perhaps much higher. Still the purpose of this study is to show the mathematical procedures in developing logistic models for the number of cases, and that procedure is accurate no matter what the numbers say.

Science will help to find a vaccine and doing such things as washing hands, covering mouths when sneezing, quarantining people for a limited time who have been exposed to the virus will certainly help. The more data that we have, the better mathematics can begin to make predictions about the future. At this point however, it really is too soon to say. We don’t know how this will end. We need more data.

But everyone should be rooting for a logistic solution with as small a carrying capacity as possible.

It is rare to have a mathematically rich (through tragic) situation evolving in front of our eyes. As the days go on and more data is available, I will continually update this article so you can see where the trends are leading.

Please be aware that I am looking at this solely from a mathematical viewpoint – seeing what trends at which the numbers point. This is not about the science and medical aspect of the disease, about which I know little other than to cover my mouth when I sneeze and wash my hands frequently.

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February 1: 24 hours later, there are now 13,801 confirmed cases of Coronavirus in Mainland China. Looking at our last 5 days of data we see:

<table>
<thead>
<tr>
<th>Date</th>
<th>Jan 28</th>
<th>Jan 29</th>
<th>Jan 30</th>
<th>Jan 31</th>
<th>Feb 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td>Cases</td>
<td>6,000</td>
<td>7,700</td>
<td>9,700</td>
<td>11,374</td>
<td>13,801</td>
</tr>
<tr>
<td>Growth</td>
<td>1,600</td>
<td>1,700</td>
<td>2,000</td>
<td>1,674</td>
<td>2,427</td>
</tr>
</tbody>
</table>

For exponential growth, we get the equation:
Cases = 190.371 \left( 1.427^{day} \right)

For logistic growth, we get the equation:
Cases = \frac{18211}{1+131.77e^{-0.4575 \text{day}}}

To the right is the graph of both equations with the exponential in green and the logistic in red.

Note the growth from January 31 to February 1 is 2,427 new cases. The growth is increasing, which is not great news. The logistic prediction now is for it to level off at just over 18,200 cases. As stringent as the measures taken in China are, they are necessary to help stem the growth of Coronavirus. The crisis is far from over.

More data tomorrow:

February 2: 48 hours later, there are now 16,634 confirmed cases of Coronavirus in Mainland China. Looking at our last 5 days of data we see:

<table>
<thead>
<tr>
<th>Date</th>
<th>Jan 29</th>
<th>Jan 30</th>
<th>Jan 31</th>
<th>Feb 1</th>
<th>Feb 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
</tr>
<tr>
<td>Cases</td>
<td>7,700</td>
<td>9,700</td>
<td>11,374</td>
<td>13,801</td>
<td>16,634</td>
</tr>
<tr>
<td>Growth</td>
<td>1,700</td>
<td>2,000</td>
<td>1,674</td>
<td>2,427</td>
<td>2,833</td>
</tr>
</tbody>
</table>

For exponential growth, we get the equation:
Cases = 204.679 \left( 1.406^{day} \right)

For logistic growth, we get the equation: Cases = \frac{22999}{1+115.38e^{-0.4014 \text{day}}}

To the right is the graph of both equations with the exponential in green and the logistic in red.

The growth from February 1 to February 2 is 2,833 new cases. The growth is still increasing, and the logistic prediction now is for it to level off at just about 23,000.
February 3: 72 hours later, there are now 19,698 confirmed cases of Coronavirus in Mainland China. Looking at our last 5 days of data we see:

<table>
<thead>
<tr>
<th>Date</th>
<th>Jan 30</th>
<th>Jan 31</th>
<th>Feb 1</th>
<th>Feb 2</th>
<th>Feb 3</th>
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<tbody>
<tr>
<td>Day</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>15</td>
</tr>
<tr>
<td>Cases</td>
<td>9,700</td>
<td>11,374</td>
<td>13,801</td>
<td>16,634</td>
<td>19,698</td>
</tr>
<tr>
<td>Growth</td>
<td>2,000</td>
<td>1,674</td>
<td>2,427</td>
<td>2,833</td>
<td>3,064</td>
</tr>
</tbody>
</table>

For exponential growth, we get the equation:

$\text{Cases} = 220.217 \left( 1.387^{\text{day}} \right)$

For logistic growth, we get the equation:

$\text{Cases} = \frac{28850}{1 + 108.498 \cdot e^{-0.361 \cdot \text{day}}}$

To the right is the graph of both equations with the exponential in green and the logistic in red.

The growth from February 2 to February 3 is 3,064 new cases. The growth is still increasing (although not by as much), and the logistic prediction now is for it to level off at just about 28,850.

Note that if cases represent the value of the function based on time, the growth represents the function’s derivative. The fact that the growth is increasing is making a statement about the function’s second derivative. The fact that the growth is increasing but by not as much is stating that the function’s 3rd derivative is negative.

February 4: 72 hours later, there are now 23,700 confirmed cases of Coronavirus in Mainland China. Looking at our last 5 days of data we see:

<table>
<thead>
<tr>
<th>Date</th>
<th>Jan 31</th>
<th>Feb 1</th>
<th>Feb 2</th>
<th>Feb 3</th>
<th>Feb 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
</tr>
<tr>
<td>Cases</td>
<td>11,374</td>
<td>13,801</td>
<td>16,634</td>
<td>19,698</td>
<td>23,700</td>
</tr>
<tr>
<td>Growth</td>
<td>1,674</td>
<td>2,427</td>
<td>2,833</td>
<td>3,064</td>
<td>4,002</td>
</tr>
</tbody>
</table>

For exponential growth, we get the equation:

$\text{Cases} = 236.140 \left( 1.370^{\text{day}} \right)$

For logistic growth, we get the equation:

$\text{Cases} = \frac{37675}{1 + 107.479 \cdot e^{-0.3212 \cdot \text{day}}}$

To the right is the graph of both equations with the exponential in green and the logistic in red.

The growth from February 3 to February 4 is 4,002 new cases. The growth is still increasing, and the logistic prediction now is for it to level off at just about 28,850.

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For each day since day 1 of this study, January 31, we have examined the projected logistic growth carrying capacity. It is increasing and close to an exponential model:

Max Cases = 11761\cdot1.256^{ \text{day} }.

If this growth of the logistic carrying capacity continues along this path, then the logistic models predict on February 11, one week from today, that the number of infected people in Mainland China would max out at just under 182,000 people. Pretty scary.

**February 5:** 72 hours later, there are now 27,767 confirmed cases of Coronavirus in Mainland China. Looking at our last 5 days of data we see:

<table>
<thead>
<tr>
<th>Date</th>
<th>Feb 1</th>
<th>Feb 2</th>
<th>Feb 3</th>
<th>Feb 4</th>
<th>Feb 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
<td>17</td>
</tr>
<tr>
<td>Cases</td>
<td>13,801</td>
<td>16,634</td>
<td>19,698</td>
<td>23,700</td>
<td>27,767</td>
</tr>
<tr>
<td>Growth</td>
<td>2,427</td>
<td>2,833</td>
<td>3,064</td>
<td>4,002</td>
<td>4,067</td>
</tr>
</tbody>
</table>

For exponential growth, we get the equation:

\[ \text{Cases} = 253.048(1.355^{ \text{day} }) \]

For logistic growth, we get the equation:

\[ \text{Cases} = \frac{46747}{1+111.079e^{-0.3968^{\text{day}}}} \]

To the right is the graph of both equations with the exponential in green and the logistic in red.

The growth from February 4 to February 5 is 4,067 new cases. The growth is still increasing although it is about the same as it was yesterday. Time will tell whether the growth will start to decrease. At this time, the logistic equation predicts that the disease will level off at 46,747 cases.
**February 6:** 96 hours later, there are now 30,923 confirmed cases of Coronavirus in Mainland China. Looking at our last 5 days of data we see:

<table>
<thead>
<tr>
<th>Date</th>
<th>Feb 2</th>
<th>Feb 3</th>
<th>Feb 4</th>
<th>Feb 5</th>
<th>Feb 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day</td>
<td>14</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
</tr>
<tr>
<td>Cases</td>
<td>16,634</td>
<td>19,698</td>
<td>23,700</td>
<td>27,767</td>
<td>30,923</td>
</tr>
<tr>
<td>Growth</td>
<td>2,833</td>
<td>3,064</td>
<td>4,002</td>
<td>4,067</td>
<td>3,247</td>
</tr>
</tbody>
</table>

For exponential growth, we get the equation:
\[
\text{Cases} = 272.207 (1.339^{\text{day}})
\]

For logistic growth, we get the equation:
\[
\text{Cases} = \frac{48442}{1 + 111.567 e^{-0.2929 \text{day}}}
\]

To the right is the graph of both equations with the exponential in green and the logistic in red.

The growth from February 5 to February 6 is 3,247 new cases. For the first time since Jan. 30, the growth has decreased somewhat and thus the logistic equation predicts that the disease will level off at 48,442 cases, not a big change from yesterday.

---

**February 7:** 120 hours later, there are now 34,075 confirmed cases of Coronavirus in Mainland China. Looking at our last 5 days of data we see:

<table>
<thead>
<tr>
<th>Date</th>
<th>Feb 3</th>
<th>Feb 4</th>
<th>Feb 5</th>
<th>Feb 6</th>
<th>Feb 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>19</td>
</tr>
<tr>
<td>Cases</td>
<td>19,698</td>
<td>23,700</td>
<td>27,767</td>
<td>30,923</td>
<td>34,075</td>
</tr>
<tr>
<td>Growth</td>
<td>3,064</td>
<td>4,002</td>
<td>4,067</td>
<td>3,247</td>
<td>3,152</td>
</tr>
</tbody>
</table>

For exponential growth, we get the equation:
\[
\text{Cases} = 293.557 (1.324^{\text{day}})
\]

For logistic growth, we get the equation:
\[
\text{Cases} = \frac{48856}{1 + 111.479 e^{-0.2918 \text{day}}}
\]

To the right is the graph of both equations with the exponential in green and the logistic in red.

The growth from February 6 to February 7 is 3,152 new cases. That is two days in a row where the growth has slowed down albeit very slightly today and thus the logistic equation predicts that the disease will level off at 48,856 cases, almost the same as yesterday.
Let’s use some calculus terms as we view the data.

\[ V: \text{ Let us define the variable } V \text{ to be the number of cases of coronavirus as a function of the day } (t \text{ for time}). \]

\[ V': \text{ Growth is the derivative of the continuous function } V \text{ which is defined as } \frac{dV}{dt}. \text{ Since we are using discrete points rather than the curve, (because we really do not have a function for } V \text{ yet, just a logistic model based on 19 points) we say that Growth is approximately equal to } V' \text{ and the growth on day } t \text{ is then defined as } V_t - V_{t-1}, t \geq 2. \]

\[ V'': \text{ Since } V' \text{ approximates the growth, } V'' \text{ approximates the change in growth and is defined as } V'_t - V'_{t-1}, t \geq 3 \]

Here are all the data points we have so far:

<table>
<thead>
<tr>
<th>Day = t</th>
<th>Jan 20</th>
<th>Jan 21</th>
<th>Jan 22</th>
<th>Jan 23</th>
<th>Jan 24</th>
<th>Jan 25</th>
<th>Jan 26</th>
<th>Jan 27</th>
<th>Jan 28</th>
<th>Jan 29</th>
<th>Jan 30</th>
<th>Jan 31</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cases = V</td>
<td>278</td>
<td>326</td>
<td>547</td>
<td>639</td>
<td>916</td>
<td>2,000</td>
<td>2,700</td>
<td>4,400</td>
<td>6,000</td>
<td>7,700</td>
<td>9,700</td>
<td>11,374</td>
</tr>
<tr>
<td>Growth = V'</td>
<td>48</td>
<td>221</td>
<td>93</td>
<td>277</td>
<td>1,084</td>
<td>700</td>
<td>1,700</td>
<td>1,600</td>
<td>2,000</td>
<td>1,674</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Change of Growth = V''</td>
<td>173</td>
<td>-128</td>
<td>184</td>
<td>807</td>
<td>-384</td>
<td>1,000</td>
<td>-1,000</td>
<td>1,000</td>
<td>300</td>
<td>-326</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Day = t</th>
<th>Feb 1</th>
<th>Feb 2</th>
<th>Feb 3</th>
<th>Feb 4</th>
<th>Feb 5</th>
<th>Feb 6</th>
<th>Feb 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cases = V</td>
<td>13,801</td>
<td>16,634</td>
<td>19,698</td>
<td>23,700</td>
<td>27,767</td>
<td>30,923</td>
<td>34,075</td>
</tr>
<tr>
<td>Growth = V'</td>
<td>2,427</td>
<td>2,833</td>
<td>3,064</td>
<td>4,002</td>
<td>4,067</td>
<td>3,237</td>
<td>3,152</td>
</tr>
<tr>
<td>Change of Growth = V''</td>
<td>753</td>
<td>406</td>
<td>231</td>
<td>938</td>
<td>65</td>
<td>-830</td>
<td>-85</td>
</tr>
</tbody>
</table>

We see that \( V \) is always positive and increasing. Obviously, we cannot have a negative number of viruses, and once we identify someone with the virus, he or she becomes someone who has or has had the virus. \( V \) encompasses the people who have had the virus and have recovered. Once you have it, you are always included in \( V \). Using calculus, we know that \( V \) is increasing because \( V' \) is always positive.

When the data collection started, \( V'' \) varied between positive and negative,. But once we got more points under our belt, \( V'' \) settled into positive numbers. We know that \( V'' \) being positive means that \( V' \) is increasing. Graphically, that means that the curve was concave up in those days.

However in the last 2 days, \( V'' \) has turned negative. That means that \( V' \) is decreasing. Graphically, that means that the curve has switched to concave down. Remember, that doesn’t mean that the number of cases is decreasing. It is still increasing but not by as much.

In the blowup of the logistic curve to the right showing the last 4 data points, you can see the subtle shift from concave up to concave down at the point second to the left. It isn’t much of a change and it is only based on the data from the past 2 days. I have read that typically the values of \( V'' \) can switch from positive to negative a number of times along the way. Eventually, though, if the logistic model holds, it will always be negative as the curve gets closer to leveling off.
As an addition to this article, let’s talk about the spread of disease. In other words, we will move away from the numbers in the logistic equation (which could apply to spread of disease as well as spread of rumors, spread of a population purchasing a new product, etc.).

\( R_0 \) (pronounce R-nought) is a mathematical term that indicates how contagious an infectious disease is. It is sometimes referred to as the reproduction number of the disease.

\( R_0 \) tells you the average number of people who will catch a disease from one contagious person. Those people are assumed to be ones who were previously free of the infection and haven’t been vaccinated (if a vaccination indeed currently exists). So if a disease has an \( R_0 \) of 10, it means that a person with the disease will transmit it to an average of 10 other people, as long as no one has been vaccinated against it or is somehow immune to it.

- If \( R_0 < 1 \), each existing infection causes fewer than one new infection. So the disease will decline and eventually die out.
- If \( R_0 = 1 \), each existing infection causes one new infection. The disease will stay alive and never die out. But there won’t be an outbreak or epidemic.
- If \( R_0 > 1 \), each existing infection cause more than one new infection. Depending on the size of \( R_0 \) and other factors, the disease will spread between people and there could be an outbreak or epidemic.

It is important to understand that when \( R_0 \) is calculated, everyone in a population is completely vulnerable to the disease. Meaning that:

- no one has been vaccinated
- no one had the disease before
- there is no way to control the spread of the disease.

When something like the coronavirus is discovered, governments take steps to control the spread of the disease (like isolation and quarantines), so \( R_0 \) is more a theoretical number that describes what would happen with the disease if there were no intervention. Back in 1918, there was an outbreak of swine flu that killed 50 million people. It was estimated that the \( R_0 \) value of that outbreak was between 1.4 and 2.8

The swine flu (H1N1 virus) came back in 2009. The \( R_0 \) value was similar, between 1.4 and 2.6, but because of vaccines and antiviral drugs, it was much less deadly.

Calculating the value of \( R_0 \) takes several factors into consideration:

First, we must have some idea of the **infectious period**, how long a time someone could spread the disease. Adults with a typical flu are usually contagious for about 8 days while with children, it is more like 2 weeks.

Second, we must have a sense of the **contact rate**. If a person who is infected with a contagious disease comes into contact with people who aren’t infected (or vaccinated, if a vaccine exists), the disease will spread more quickly. If the infected person stays at home or in a hospital, or quarantined somehow, the disease will spread more slowly. The higher the contact rate, in general the higher the \( R_0 \) value. That is why some ships have been quarantined currently and people not allowed to get off through the infectious period.
To date, on the cruise ship Diamond Princess with over 3,000 passengers and crew, 61 people have been tested positive for Coronavirus. Those people have had the ability of infecting anyone on that ship. Those people were taken off the ship, but if they stayed on, the worst-case is that they infect everyone on the ship. That is horrifying, but it would be far worse if those 61 people had begun to spread it to anyone they come into contact with on the outside. Quarantining, as frightening and inconvenient as it is for those people on the ship, makes it very unlikely that any of those people can spread the disease once they leave the ship.

A common cold is infectious for about 2 weeks. When I taught I had about 5 or 6 colds a year. It was no wonder with all that contact with kids. When I retired to Florida, I have had 2 colds over a period of 5 years.

Third, the mechanism of transmission must be determined. Disease that can travel through the air such as the flu or measles spread very quickly. It isn’t just physical contact. Simply being in the same room and breathing the same air can give you the flu, even without any contact.

Disease that are transmitted through bodily fluids (such as Ebola or HIV) are not as easy to catch or spread. Spread only occurs when coming into contact with infected blood, saliva or other bodily fluids. So despite these diseases being thought of as deadly, they have a smaller $R_0$ than disease that are airborne.

Following are the approximate values of $R_0$ for common infectious diseases.

<table>
<thead>
<tr>
<th>Disease</th>
<th>Transmission</th>
<th>$R_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measles</td>
<td>Airborne</td>
<td>12-18</td>
</tr>
<tr>
<td>Diphtheria</td>
<td>Saliva</td>
<td>6-7</td>
</tr>
<tr>
<td>Smallpox</td>
<td>Airborne</td>
<td>5-7</td>
</tr>
<tr>
<td>Polio</td>
<td>Fecal-oral</td>
<td>5-7</td>
</tr>
<tr>
<td>Rubella</td>
<td>Airborne</td>
<td>5-7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Disease</th>
<th>Transmission</th>
<th>$R_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mumps</td>
<td>Airborne</td>
<td>4-7</td>
</tr>
<tr>
<td>HIV/AIDS</td>
<td>Sexual contact</td>
<td>2-5</td>
</tr>
<tr>
<td>SARS</td>
<td>Airborne</td>
<td>2-5</td>
</tr>
<tr>
<td>1918 flu</td>
<td>Airborne</td>
<td>2-3</td>
</tr>
<tr>
<td>Ebola</td>
<td>Body fluids</td>
<td>1.5-2.5</td>
</tr>
</tbody>
</table>

It is believed that the Coronavirus virus has airborne transmission and has a $R_0$ value of 1.4 - 3.9. Do not be deceived by the fact that the values are just slightly above 1. The 1918 flu had an $R_0$ value of 2 - 3 and killed 50 million people.

$R_0$ is difficult to find and is a theoretical number. There are a number of methods used to approximate $R_0$ and each will give different values. In the table above, not all the values of $R_0$ came from the same method, so they should be used with caution when comparing. One method of approximating $R_0$ uses matrices, showing transitions from one matrix to another. Not usually taught in high school, this stochastic process method is covered in my Matrices manual on the MasterMathMentor.com website.

http://www.mastermathmentor.com/mmm/Matrices.ashx

To understand the concept of $R_0$ better than I can explain it, I urge you to look at these two wonderful Khan Academy videos presented by Dr. Rishi Desai, a pediatric infectious disease physician and former epidemiologist with the Centers for Disease Control and Prevention (CDC)


www.mastermathmentor.com
February 8: 144 hours later, there are now 36,767 confirmed cases of Coronavirus in Mainland China. Looking at our last 5 days of data we see:

<table>
<thead>
<tr>
<th>Date</th>
<th>Feb 4</th>
<th>Feb 5</th>
<th>Feb 6</th>
<th>Feb 7</th>
<th>Feb 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>20</td>
</tr>
<tr>
<td>Cases</td>
<td>23,700</td>
<td>27,767</td>
<td>30,923</td>
<td>34,075</td>
<td>36,767</td>
</tr>
<tr>
<td>Growth</td>
<td>4,002</td>
<td>4,067</td>
<td>3,247</td>
<td>3,152</td>
<td>2,692</td>
</tr>
</tbody>
</table>

For exponential growth, we get the equation:
\[
\text{Cases} = 317.441 \left(1.309^{\text{day}}\right)
\]

For logistic growth, we get the equation:
\[
\text{Cases} = \frac{48648}{1+111.664e^{-0.2925\text{day}}}
\]

To the right is the graph of both equations with the exponential in green and the logistic in red.

The growth from February 7 to February 8 is 2,692 new cases. That is now three days in a row where the growth has slowed down and thus the logistic equation predicts that the disease will level off at 48,648 cases, actually down yesterday.

Has the curve already passed its inflection point and the rate of growth starting to slow? Only time will tell. But this is the opinion of Dr. Robert Siegel, professor of microbiology and immunology at Stanford University: [https://www.foxnews.com/opinion/dr-robert-siegel-coronavirus-epidemic-could-be-contained-in-months-global-pandemic-unlikely](https://www.foxnews.com/opinion/dr-robert-siegel-coronavirus-epidemic-could-be-contained-in-months-global-pandemic-unlikely)

The most encouraging news comes from looking at the rate of increase in the epidemic – the shape of the epidemic curve. It appears that a turning point (the so-called “inflection point”) has already been reached. This is the place where the curve starts to flatten out, signaling that control may be in sight.

But like the battle against a fire on its way to containment, it is critical to keep the epidemic from gaining a stronghold in a new continent.

For example, as of Thursday there was an 11 percent daily increase in the number of cases. But this rate of increase is less than the day before and much less than a week earlier.

This drop has occurred even though the level of scrutiny for new cases of the disease has increased.

I am going to repeat this one more time: This is based on the numbers from the Johns Hopkins Center for Systems Science and Engineering. They get their data from the World Health Organization. [https://www.who.int/emergencies/diseases/novel-coronavirus-2019](https://www.who.int/emergencies/diseases/novel-coronavirus-2019)

It is hard to say how accurate it is. The Center for Disease Control and Prevention [https://www.cdc.gov/coronavirus/2019-ncov/index.html](https://www.cdc.gov/coronavirus/2019-ncov/index.html) has not been permitted into China. There are areas within Mainland China that are quite remote, and it is not likely that any kind of testing has been done there. So the actual figures of coronavirus cases may be higher, perhaps much higher. Still the purpose of this study is to show the mathematical procedures in developing logistic models for the number of cases, and that procedure is accurate no matter what the numbers say.
February 9: 1 week later, there are now 39,790 confirmed cases of Coronavirus in Mainland China. Looking at our last 5 days of data we see:

<table>
<thead>
<tr>
<th>Date</th>
<th>Feb 5</th>
<th>Feb 6</th>
<th>Feb 7</th>
<th>Feb 8</th>
<th>Feb 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>20</td>
<td>21</td>
</tr>
<tr>
<td>Cases</td>
<td>27,767</td>
<td>30,923</td>
<td>34,075</td>
<td>36,767</td>
<td>39,790</td>
</tr>
<tr>
<td>Growth</td>
<td>4,067</td>
<td>3,247</td>
<td>3,152</td>
<td>2,692</td>
<td>3,023</td>
</tr>
</tbody>
</table>

For logistic growth, we get the equation: \( \text{Cases} = \frac{49918}{1 + 109.665e^{-0.2879 \cdot \text{day}}} \)

To the right is the graph the logistic equation in red. I have graphed the projected carrying capacities for the past 11 days in blue. Right now, creating a logistic curve from all the data makes the maximum carrying capacity level out just below 53,000 cases.

The growth from February 8 to February 9 is 3,023 new cases, up a bit from yesterday. Just as the growth figures occasionally decreased as the virus was initially spreading, they can also increase slightly while the general pattern is decreasing. More time is needed.

February 10: In terms of people who have recovered, as of February 10, 3,278 people have been said to recover from the coronavirus. I am not sure how this figure has been arrived at. They must have been diagnosed with the virus, treated for it, and then a determination was made that they no longer had it. But worldwide, over 40,000 cases have been reported with approximately 1,000 deaths. With the known recovered, that leaves about 90% unaccounted for. Obviously once someone gets the virus, recovery does not happen overnight so recovery data will always be behind known case data. Since there is no drug available to treat the virus, many hope to get better on their own while quarantining themselves so they do not spread it. One would have to be tested again to be sure he is no longer infected. This takes a lot of time, so the recovery numbers are quite incomplete.

Still, from a mathematical point of view, the numbers of people reported recovered also appears to have a logistic pattern to it, shown in green. But, the exponential model in red is not far away from the majority of known points. Since the logistic model has a carrying capacity of just over 6,000 people and we hope that most people with the virus will recover eventually, the data is woefully incomplete.

To better explain that mathematically, suppose we have data with four points \((1, 1.1), (2, 2.0), (3, 3.2), (4, 4.5)\). We graph them, perform linear regression, and the line does an admirable job of describing the relationship. It is safe to say that if we wanted the value of \(y\) at \(x = 2.5\), we could confidently use the line as a predictor. However there are other regression models that can be used as shown in the second figure: quadratic in red, cubic in green, exponential in orange, and power in brown. Again, if we wanted the value of \(y\) at some value of \(x\) well away from the known points (like \(x = 10\)), we will get wildly different answers. Which is correct? Who knows? More data is needed.
February 10: 8 days later, there are now 42,306 confirmed cases of Coronavirus in Mainland China. Looking at our last 5 days of data we see:

<table>
<thead>
<tr>
<th>Date</th>
<th>Feb 6</th>
<th>Feb 7</th>
<th>Feb 8</th>
<th>Feb 9</th>
<th>Feb 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day</td>
<td>18</td>
<td>19</td>
<td>20</td>
<td>21</td>
<td>22</td>
</tr>
<tr>
<td>Cases</td>
<td>30,923</td>
<td>34,075</td>
<td>36,767</td>
<td>39,790</td>
<td>42,306</td>
</tr>
<tr>
<td>Growth</td>
<td>3,247</td>
<td>3,152</td>
<td>2,692</td>
<td>3,023</td>
<td>2,516</td>
</tr>
</tbody>
</table>

For logistic growth, we get the equation:

\[
\text{Cases} = \frac{50625}{1 + 108.145e^{-0.2852 \text{ day}}} 
\]

To the right is the graph the logistic equation in red. I have graphed the projected carrying capacities for the past 12 days in blue. Right now, creating a logistic curve from all the data makes the maximum carrying capacity level out just above 52,000 cases.

The growth from February 9 to February 10 is 2,516 new cases, down 17% from yesterday. I heard on the news a reporter saying the number of cases is now stable. Nothing could be further from the truth. Stable means not changing. What was meant was that the growth is closer to stable. The last 5 days, the growth has been in that 2,500 to 3,000 area. Certainly a difference between those two figures, but when we are talking about Mainland China with a population of close to 1.4 billion people, that difference of 500 is miniscule. Since 4 of those 5 days, the growth has been decreasing, the red cases curve above appears to be past its inflection point and concave down. Still, many more days are needed to make this claim.

February 11: 9 days later, there are now 44,311 confirmed cases of Coronavirus in Mainland China. Looking at our last 5 days of data we see:

<table>
<thead>
<tr>
<th>Date</th>
<th>Feb 7</th>
<th>Feb 8</th>
<th>Feb 9</th>
<th>Feb 10</th>
<th>Feb 11</th>
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<tr>
<td>Day</td>
<td>19</td>
<td>20</td>
<td>21</td>
<td>22</td>
<td>23</td>
</tr>
<tr>
<td>Cases</td>
<td>34,075</td>
<td>36,767</td>
<td>39,790</td>
<td>42,306</td>
<td>44,311</td>
</tr>
<tr>
<td>Growth</td>
<td>3,152</td>
<td>2,692</td>
<td>3,023</td>
<td>2,516</td>
<td>2,005</td>
</tr>
</tbody>
</table>

For logistic growth, we get the equation:

\[
\text{Cases} = \frac{51076}{1 + 108.879e^{-0.2832 \text{ day}}} 
\]

Another day where the growth decreased. To the right is the graph the logistic equation in red. It appears that we have passed the point of inflection and the cases are leveling off. That presupposes that the data collection methods stay the same. With the World Health Organization now in China, things could change. I have graphed the projected carrying capacities for the past 13 days in blue. Right now, creating a logistic curve from all the data makes the maximum carrying capacity level out just below 52,000 cases.
February 12: I was getting ready to do my 7:00 PM update. I looked at the data around 6 PM and found that the number of new cases today had jumped only a few hundred. When I got set to actually do it though, I checked the actual numbers and found to my shock that the new cases had jumped over 14,000. What had happened?

It took a few hours to understand what had occurred as many news sources had not picked up on it. Many sources were claiming that the growth was still decreasing. Finally, at 9:20 PM I read: “The huge rise in confirmed cases comes from a tweak in how the Chinese authorities are tallying infections. The government is now including ‘clinically diagnosed cases’ i.e. people diagnosed on the basis of their symptoms rather than testing positive in order to make it easier for those patients to access treatment. 13,332 of the new cases fall under that classification.”

So nothing has really changed other than how we administratively define a case. However, since we are looking at the cases strictly from a numerical point of view, all of a sudden, the rules changed. Yes, we can add the new data to our existing data and recalculate the logistic curve. But that would be incorrect mathematically because the definition of a case has now changed.

This allows us to use a method that is taught in most precalculus courses and above, the piecewise function. We use piecewise functions to give several rules for the number of cases based on time. Since this new definition changed on day 24, we will define our case equation as a piecewise function with $t < 24$ and $t \geq 24$. Obviously, we only have one data point now (24, 59,789).

\[
\text{Cases} = \begin{cases} 
51076 & t < 24 \\
1 + 108.879e^{-0.2832t} & t \geq 24 
\end{cases}
\]

Over the next few days, we will see if the growth still continues to be decreasing despite this big spike on day 24. As I mentioned, this ongoing problem, as dangerous and scary as it is, is a wealth of real-life uses of mathematics.

February 13, 14, and 15

On February 13th, the total cases were about 62,900 and on the 14th, 66,292, That’s an increase of over 3,000 each day, much more than before the 13th. However with the new rules defining a case, that doesn’t necessarily indicate a big change from what was occurring before the new classification. If someone has a cough or cold and a chest X-ray showing chest infection, they now fall into the category of the coronavirus. On Feb. 15, the number of cases were at 68,407, an increase of 2,115 new cases, down from yesterday. We will wait several days before we start to look for patterns. Right now, it is quite linear with $r = 0.991$. 
The recovery data curve has been updated. It took quite a while for the TI-84 calculator to compute a logistic curve because there is no visual evidence that the curve has passed its inflection point. The rate of recovery is still increasing which is good news. The model shows that approximately 23,000 will recover from it. Considering that there are currently 68,000 cases, we know this number is going to climb dramatically with “only” 1,666 deaths. It will be months, if ever, before the recovery data will be even nearly complete.

February 16: 4 weeks since data has been kept, there are now 70,526 confirmed cases of Coronavirus in Mainland China. Looking at our last 5 days of data we see:

<table>
<thead>
<tr>
<th>Date</th>
<th>Feb 12</th>
<th>Feb 13</th>
<th>Feb 14</th>
<th>Feb 15</th>
<th>Feb 16</th>
</tr>
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<tr>
<td>Day</td>
<td>24</td>
<td>25</td>
<td>26</td>
<td>27</td>
<td>28</td>
</tr>
<tr>
<td>Cases</td>
<td>59,789</td>
<td>62,900</td>
<td>66,292</td>
<td>68,407</td>
<td>70,526</td>
</tr>
<tr>
<td>Growth</td>
<td>3,111</td>
<td>3,392</td>
<td>2,115</td>
<td>1,849</td>
<td>1,849</td>
</tr>
</tbody>
</table>

For logistic growth, we get the equation:

\[ \text{Cases} = \frac{78498}{1 + 145.708e^{-0.2558 \text{day}}} \]

Since the new classification, the change in the new cases have decreased below the point that they were before the less inclusive old classification. There are only 5 data points in this set, but it appears that the graph is starting to level off, assuming the data is accurate. Time will tell.

February 17: Increase in new cases just about the same as yesterday. Since the growth had been dropping, the carrying capacity inched up by about 1,000.

\[ \text{Cases} = \frac{79410}{1 + 110.497e^{-0.2423 \text{day}}} \]

<table>
<thead>
<tr>
<th>Date</th>
<th>Feb 13</th>
<th>Feb 14</th>
<th>Feb 15</th>
<th>Feb 16</th>
<th>Feb 17</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day</td>
<td>25</td>
<td>26</td>
<td>27</td>
<td>28</td>
<td>29</td>
</tr>
<tr>
<td>Cases</td>
<td>62,900</td>
<td>66,292</td>
<td>68,407</td>
<td>70,526</td>
<td>72,364</td>
</tr>
<tr>
<td>Growth</td>
<td>3,111</td>
<td>3,392</td>
<td>2,115</td>
<td>1,849</td>
<td>1,838</td>
</tr>
</tbody>
</table>
February 18: It’s been a full month since data had first been kept. The increase in new cases slightly down from yesterday. Since the growth has been fairly constant the last few days, the carrying capacity is still going up slightly. Now it is just under 81,000.

\[
\text{Cases} = \frac{80971}{1 + 71.678e^{-0.2213 \text{day}}}
\]

<table>
<thead>
<tr>
<th>Date</th>
<th>Feb 14</th>
<th>Feb 15</th>
<th>Feb 16</th>
<th>Feb 17</th>
<th>Feb 18</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day</td>
<td>26</td>
<td>27</td>
<td>28</td>
<td>29</td>
<td>30</td>
</tr>
<tr>
<td>Cases</td>
<td>66,292</td>
<td>68,407</td>
<td>70,526</td>
<td>72,364</td>
<td>74,142</td>
</tr>
<tr>
<td>Growth</td>
<td>3,392</td>
<td>2,115</td>
<td>1,849</td>
<td>1,838</td>
<td>1,778</td>
</tr>
</tbody>
</table>

I have held out looking at deaths from coronavirus, but they have increased to a point where the numbers are sadly, following a pattern as well. To the right are the number of worldwide deaths from coronavirus from January 23 through February 18. It has a logistic look and it is uncertain whether or not the inflection point has been reached although the deaths have stayed fairly constant the last several days.

\[
\text{Deaths} = \frac{2670}{1 + 48.309e^{-0.18763 \text{day}}}
\]

With this model, the carrying capacity is 2,789 deaths and it will probably take several weeks before the numbers get close to that figure.

The curve for recoveries seems to be somewhere between exponential and logistic. Eventually, it will be logistic (there will be a finite number of people who recover from the virus), but this is one curve we hope keeps climbing instead of leveling out. Right now, it is modeled to level out at 28,254 recoveries.
**February 20:** The Chinese changed the rules again for defining a case of Coronavirus. On February 12, the definition of a case became more inclusive and there was a huge jump in cases, although they settled down to what appeared to be a logistic model. Now, the definition has changed again, seeing to make the guidelines for coronavirus less inclusive. So the number of new cases has dropped quite a bit. “Every time you change the case definition, that then means you have a reset in terms of what you’re actually looking at,” said Michael Osterholm, director of the University of Minnesota’s Center for Infectious Diseases Research and Policy. “I think between the inability to determine the actual number of people infected and how cases are now being called a case means at best you can get trend data, possibly, but not more than that.”

**February 23:** 3 days later and I am not sure what to make of the data. According to the Johns Hopkins website, there were only 10 new cases in Mainland China over the past 24 hours. That’s kind of hard to believe.

I will no longer show the equations and will just briefly comment on the new data. The carrying capacity is shown by the dotted line at the top of the graphs. No equation is placed on the new cases since the new classification has been used because of the lack of data and questions about its reliability.

**February 27:** With the stock market tanking and a lot of scary predictions on the Internet, the one good piece of news is that it seems that the situation in mainland China is stabilizing a bit. Cases are growing at about 500 a day compared to 3,000 about 3 weeks ago. The recovery curve doesn’t appear to have hit its inflection point yet, meaning that its growth is still somewhat exponential. And the death curve’s logistic carrying capacity went down for the first time.
March 1: The stock markets are taking a huge hit and there have been a death in America as well as new cases. But the growth in Mainland China appears to be slowing. Since the classification changed for the second time, the logistic graph is now appearing to level off at just above 84,000 cases. The recovery graph seems to not have reached its inflection point yet while the deaths seem to be leveling off as well. This all appears to be good news, compared to a week ago.

But other countries where the virus now resides now will have to go through this type of growth, hopefully to a much lesser degree.

A dramatic image from NASA shows the amount of pollution over China in the form of nitrogen dioxide emitted by motor vehicles and industrial facilities in January as opposed to February. NASA noted that China's Lunar New Year celebrations in late January and early February have been linked to decreases in pollution levels in the past. But it said they normally increase once the celebrations are over. This year, it hasn’t.

I guess this is good news.

Let’s discuss the death rate. I saw an article from the Cato Institute on the misleading arithmetic of coronavirus death rates. I will quote from the article and put things in my words to fit my audience but I give full credit to the website: https://www.cato.org/blog/misleading-arithmetic-covid-19-death-rates

Currently, the percentage of infected people who die from the disease (the death rate) is much lower than the 2 - 4% estimates commonly reported. That is because the number of infected people is much larger than the number tested and reported.

The triangle graph, from a February 10 study from Imperial College, London, shows that most people infected by COFID-19 (the coronavirus) are never counted as being infected. That is because the bottom of the pyramid represents the largest population of those infected with either mild, non-specific symptoms, or who are asymptomatic. Several studies have found that 80% of all COVID-19 cases have relatively minor symptoms which end without severe illness and therefore remain unreported.
A Chinese study in the Journal of the American Medical Association, February 20, found a “case-fatality rate” of 2.3%, meaning 1,023 died out of 44,672 cases. But the total sample in that study (72,314) included an additional 27,642 non-confirmed cases, and simply including those cases (let alone unreported minor cases) would have reduced the fatality rate to 1.4%. China-based estimates are largely confined to cases with the most severe symptoms, so it should be no surprise that the reported death rate among severe cases is much higher than it would be if the data also included “most people” who “have a mild disease and get better.”

On February 28, the Director General of WHO reported that “Outside China, there are now 4,351 cases in 49 countries, and 67 deaths.” Deaths of 67 divided by 4,351 seems to demonstrate a death rate of 1.5%. But such calculations are highly misleading. They assume the denominator of that ratio (4,351) is as accurate as the numerator (67). Yet, people with “mild cases who get better” are unlikely to ever be included in the denominator.

If the WHO estimate of 4,351 confirmed cases amounted to 30% of the actual number infected outside of China at that time, for example, then the combined total of both unreported and confirmed cases would be 4,351 divided by 0.30 or 14,503. In that case, the actual death rate would be 67 divided by 14,503, or less than one half of one percent (0.46%).

Let’s use the current statistics to see how that would work. As of Tuesday, March 3, there are 92,303 confirmed cases of coronavirus in the world. There are 3,131 deaths attributed to it. The death rate is therefore estimated to be \( \frac{3131}{92303} \approx 3.39\% \). However if the estimate of confirmed cases amounts to 30% of the actual number of people infected (meaning that 70% of people with coronavirus have mild symptoms, don’t even report it, and get better), then

\[
\text{confirmed cases} = 0.3 \left( \text{people having COVID-19} \right)
\]

\[
\text{people having COVID-19} = \frac{\text{confirmed cases}}{0.3} = \frac{92303}{0.3} = 307,677
\]

So the death rate would be \( \frac{3131}{307677} \approx 1.02\% \)

If the estimate of confirmed cases amounts to 20% of the actual number of people infected (meaning that 80% of people with coronavirus have mild symptoms, don’t even report it, and get better), then

\[
\text{confirmed cases} = 0.2 \left( \text{people having COVID-19} \right)
\]

\[
\text{people having COVID-19} = \frac{\text{confirmed cases}}{0.2} = \frac{92303}{0.2} = 461,515
\]

So the death rate would be a much smaller \( \frac{3131}{460985} \approx 0.07\% \)

This article deals with mathematics and thus deals with numbers. Numbers can be made to say anything. No one knows the actual number of people who are or will be infected by the coronavirus. By the nature of its definition, no one will ever know. But the fact that many people apparently get very mild symptoms of coronavirus and thus go unreported in the statistical information that is amassed tells us that the death rate is certainly less than what is being reported.

And to subtly change the quote Jordan Ellenberg, author of How Not To Be Wrong: The Power of Mathematical Thinking, “to divide two numbers is merely computation. To decide what the denominator is in the division process is mathematics!”

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March 4: There is little doubt the cases in Mainland China are increasing at a slower rate. The derivative of the logistic curve at $t = 45$ is about 234 meaning that the cases are growing at about 234 cases per day. At the height of the outbreak on February 14, it was growing by about 3,400 cases per day. The recovery curve continues to increase steadily. However, the deaths from coronavirus after appearing to level off, have increased at a greater rate. This is most likely because that deaths from countries outside China are now starting to accumulate. At a certain point in time, it may be necessary to split the deaths curve into a piecewise function.

Of more importance now is the growth of Coronavirus outside Mainland China. Below is the graph of the data points (day, cases). It is impossible to even begin to fit a logistic curve to it as it hasn’t gotten close to reaching its inflection point. The scary exponential graph is shown. If the trend continues in China, I will no longer show its growth and switch to this growth of Coronavirus outside of China. I will continue to show the recovery graph which should go up even more dramatically as the number of cases increases. I will also show the graph of deaths but split it into two pieces, when Mainland China was the main contributor and then when the outside world had more deaths.

March 6: Based on the data, it appears that the growth of coronavirus in Mainland China is on the wane. It certainly can rise again as more people go back to work and start to interact again. But for now, it is time to concentrate on the rest of the world and particularly the United States. Part 2 of this blog will do so.

You can download it at: [http://www.mastermathmentor.com/mmm-archive/CoronaVirus2.pdf](http://www.mastermathmentor.com/mmm-archive/CoronaVirus2.pdf)