Solutions – Jeopardy Sample

Go Straight 100

What is C?

$$s(t)=\frac{1}{2}e^{2t}-t^2+C$$

$$s(0)=\frac{1}{2}+C=0\Rightarrow C=-\frac{1}{2}$$

$$s(t)=\frac{1}{2}e^{2t}-t^2-\frac{1}{2}$$

$$s(1)=\frac{e^2-3}{2}$$

Go Straight 200

What is B?

Say at $t=1$,the particle is at $y=0$

At $t=2$, the particle is at $y=-9$

At $t=3$, the particle is at $y=-8$

At $t=5$, the particle is at $y=4$ so it passed $y=0$

At $t=6$, the particle is at $y=-4$ so it passed $y=0$

Go Straight 300

What is D?

$$v\left(\frac{1}{4}\right)=\frac{-\sin\left(\frac{\pi}{2}\right)}{\frac{1}{4}}=-4 : \text{Particle moving left}$$

$$a(t)=\frac{-2t\pi\cos(2t\pi)+\sin(2t\pi)}{t^2}$$

$$a\left(\frac{1}{4}\right)=\frac{-2t\pi\cos\left(\frac{\pi}{2}\right)+\sin\left(\frac{\pi}{2}\right)}{\frac{1}{16}}=16 : \text{Acceleration to the right}$$

So particle is slowing down

Go Straight 400

What is 335 miles?

$$\int_0^{12} |v(t)|\,dt \approx 2\frac{|v(0)|+|v(2)|}{2} + 3\frac{|v(2)|+|v(5)|}{2} + 4\frac{|v(5)|+|v(9)|}{2} + 3\frac{|v(9)|+|v(12)|}{2}$$

$$\int_0^{12} |v(t)|\,dt \approx 2\frac{20+50}{2} + 3\frac{50+20}{2} + 4\frac{20+30}{2} + 3\frac{30+10}{2}$$

$$\int_0^{12} |v(t)|\,dt \approx 70+105+100+60 = 335 \text{ miles}$$

Go Straight 500

What is E?

$$v(t) = e^{-t}(\cos t + \sin t) - e^{-t}(\sin t - \cos t)$$

$$= 2e^{-t}\cos t = 0$$

$$t = \frac{\pi}{2}, \frac{3\pi}{2}$$

t	$x(t)$
0	$e^0(-1) = -1$ (furthest left)
$\frac{\pi}{2}$	$e^{-\pi/2}(1) = e^{-\pi/2} = .208$ (furthest right)
$\frac{3\pi}{2}$	$e^{-3\pi/2}(-1) = -e^{-3\pi/2} = -.009$
2π	$e^{-2\pi}(1) = e^{-2\pi} = .002$

Distance $= .208 - (-1) = 1.208$

Things Change 100

What is B?

$$V = \pi r^2 h$$

$$V = 400\pi h$$

$$\frac{dV}{dt} = 400\pi\frac{dh}{dt}$$

$$1000(.133) = 400\pi\frac{dh}{dt}$$

$$\frac{dh}{dt} = .106\frac{\text{ft}}{\text{min}}$$

The 8 feet doesn't enter into the problem as the height increases at a constant rate since the sides of the cylinder are vertical.

Things Change 200

What is 490?

People entering:

$$2\left(\frac{380+450}{2}\right)+1\left(\frac{450+200}{2}\right)+2\left(\frac{200+30}{2}\right)=1385$$

People leaving:

$$2\left(\frac{100+80}{2}\right)+1\left(\frac{80+420}{2}\right)+2\left(\frac{420+620}{2}\right)=1470$$

$$575+1385-1470=490$$

Things Change 300

What is A?

$$x=4,\frac{dx}{dt}=-40,y=3,\frac{dy}{dt}=20$$

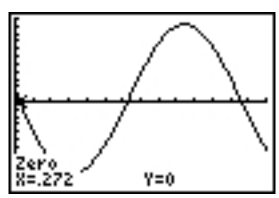
$$d=\sqrt{x^2+y^2}$$

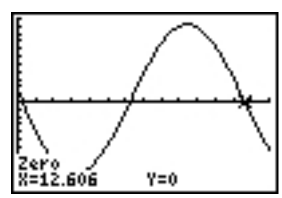
$$\frac{dd}{dt}=\frac{1}{2\sqrt{x^2+y^2}}\left(2x\frac{dx}{dt}+2y\frac{dy}{dt}\right)=\frac{x\frac{dx}{dt}+y\frac{dy}{dt}}{\sqrt{x^2+y^2}}$$

$$\frac{dd}{dt}=\frac{4(-40)+3(20)}{\sqrt{40^2+30^2}}=\frac{-100}{50}=-2$$

Things Change 400

What is E?





$$f'(t)=(2t)^{1/2}+20\cos(.5t)-15$$

$$f''(t)=\frac{1}{(2t)^{1/2}}-10\sin(.5t)=0$$

The graph of $f''(t)$ shows that there are two possibilities for the maximum rate of change at $t=.272$ and $t=12.606$. We also must examine the endpoints.

$$f'(0)=5$$

$$f'(.272)=5.553$$

$$f'(12.606)=10.017$$

$$f'(14)=5.370$$

The maximum occurs when $t=12.606$ which corresponds to year 2002.

Things Change 500

What is B?

$$V = s^3$$
$$\frac{dV}{dt} = 3s^2\frac{ds}{dt} = ks$$
$$\int 3s\ ds = k\ dt$$
$$\frac{3s^2}{2} = kt + C$$
$$3s^2 = 2kt + C \Rightarrow 3(4) = C \Rightarrow C = 12$$
$$3s^2 = 2kt + 12 \Rightarrow 3(16) = 6k + 12 \Rightarrow k = 6$$
$$3s^2 = 12t + 12 \Rightarrow s^2 = 4(t+1)$$
$$s = 2\sqrt{t+1}$$
$$s(8) = 6$$

Hit the Slopes – 100

What is E?

$$y = \sin^{-1}(e - x)$$
$$y' = \frac{-1}{\sqrt{1-(e-x)^2}} \Rightarrow y'(e) = \boxed{y'(e) = -1}$$
$$y = \frac{x^3+1}{x+1} = \frac{(x+1)(x^2-x-1)}{x+1} = x^2 - x - 1$$
$$y' = 2x - 1 \Rightarrow \boxed{y'(e) = 2e - 1}$$
$$y = \ln\left(\frac{2x+e}{x+2e}\right)^2 = 2\left[\ln(2x+e) - \ln(x+2e)\right]$$
$$y' = 2\left[\frac{2}{2x+e} - \frac{1}{x+2e}\right] \Rightarrow \boxed{y'(e) = 2\left[\frac{2}{3e} - \frac{1}{3e}\right] = \frac{2}{3e}}$$

Hit the Slopes – 200

What is B?

At $x = 0$, $y' \approx 1$, eliminating A. For larger values of x the slope is negative and C, D, E would all have positive slopes.

Hit the Slopes – 300

What is D?

$$y = f(x)g(x) \Rightarrow y' = f(x)g'(x) + g(x)f'(x)$$

$$y'(1) = f(1)g'(1) + g(1)f'(1) = 4(-2) + 2(3) = \boxed{-2}$$

$$y = \frac{1}{f^2(x)} = f^{-2}(x) \Rightarrow y' = \frac{-2}{f^3(x)}$$

$$y'(1) = \frac{-2}{4^3} = \boxed{\frac{-1}{32}}$$

$$y = \sqrt{f(x) + g(x)} \Rightarrow y' = \frac{f'(x) + g'(x)}{2\sqrt{f(x) + g(x)}}$$

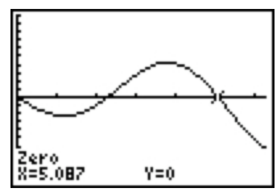
$$y'(1) = \frac{f'(1) + g'(1)}{2\sqrt{f(1) + g(1)}} = \frac{3-2}{2\sqrt{4+2}} = \boxed{\frac{1}{2\sqrt{6}}}$$

$$y = f(g(x)) \Rightarrow y' = f'(g(x)) \cdot g'(x)$$

$$y'(1) = f'(g(1)) \cdot g'(1) = f'(2)(-2) = 2(-2) = \boxed{-4}$$

Hit the Slopes – 400

What is A?



$$y' = x(-\sin x) + \cos x$$

$$y'' = x(-\cos x) - \sin x - \sin x = -x\cos x - 2\sin x$$

The maximum slope must occur where $y'' = 0$ and y'' switches from positive to negative. This is at 5.087. $y(5.087) = 1.861$.

Note : $y'(0) = 1, y'(2\pi) = 1$

Hit the Slopes – 500

What is D?

$$\frac{dy}{dx} = \frac{-2x}{e^{2y}} \Rightarrow e^{2y}dy = -2xdx$$

$$\frac{1}{2}e^{2y} = -x^2 + C$$

$$\frac{1}{2} = C \Rightarrow \frac{1}{2}e^{2y} = \frac{1}{2} - x^2 \Rightarrow e^{2y} = 1 - 2x^2$$

$$2y = \ln(1-2x^2) \Rightarrow y = \frac{\ln(1-2x^2)}{2}$$

$$\text{Domain}: 1 - 2x^2 > 0 \Rightarrow 2x^2 < 1 \Rightarrow x^2 < \frac{1}{2} \Rightarrow \boxed{-\frac{\sqrt{2}}{2} < x < \frac{\sqrt{2}}{2}}$$

