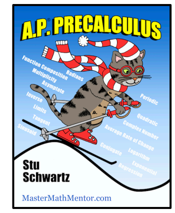
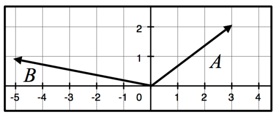
Topic 4.6 – Linear Transformations & Matrices – Classwork



We have looked at matrix multiplication but gave no real reason behind its rules. We will now look at a geometric application of matrix multiplication that is used in every video game that you play. Here we think of matrices as a way of transforming vectors – starting with an input vector and ending with an output vector. So when we have a vector in the form of $\langle a,b\rangle$ or $a\mathbf{i}+b\mathbf{j}$, it is convenient to think of it as a column matrix $\begin{bmatrix} a \\ b \end{bmatrix}$.

Suppose we have the two vectors $A=\langle 3,2\rangle$ and $B=\langle -5,1\rangle$. We will write them as $A=\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ and $B=\begin{bmatrix} -5 \\ 1 \end{bmatrix}$.

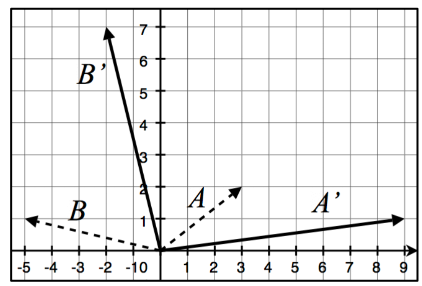


We show these two vectors graphically to the left.

Now suppose we wish to modify these vectors by matrix $M=\begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}$. We look at

$$A' = MA=\begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \end{bmatrix}=\begin{bmatrix} 1(3)+3(2) \\ -1(3)+2(2) \end{bmatrix}=\begin{bmatrix} 9 \\ 1 \end{bmatrix}$$

$$B' = MB=\begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} -5 \\ 1 \end{bmatrix}=\begin{bmatrix} 1(-5)+3(1) \\ -1(-5)+2(1) \end{bmatrix}=\begin{bmatrix} -2 \\ 7 \end{bmatrix}$$



To the right, we see how the vectors have been transformed. It appears that the two vectors have been lengthened and rotated to the right.

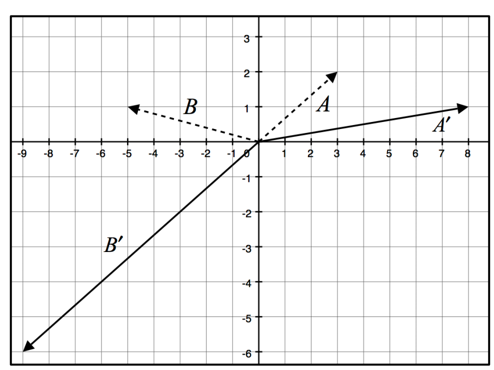
We have performed a *linear transformation* on *A* and *B*. By that we mean that the line creating the original vector is still a line but its length has been changed and it has been rotated about the origin. – starting with an input vector and ending with an output vector.

Let's change our modification matrix to $M=\begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix}$ and rather working with two column vectors *A* and *B*, lets combined them into one 2 by 2 matrix.

$$\begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ 2 & 1 \end{bmatrix}=\begin{bmatrix} 8 & -9 \\ 1 & -6 \end{bmatrix}$$

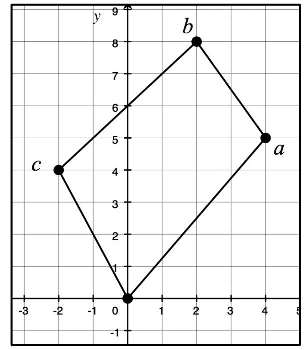
So vector *A* has been transformed to $\begin{bmatrix} 8 \\ 1 \end{bmatrix}$

And vector *B* has been transformed to $\begin{bmatrix} -9 \\ -6 \end{bmatrix}$



The two vectors have been lengthened and reflected somewhat about the origin in a manner that isn't clear.

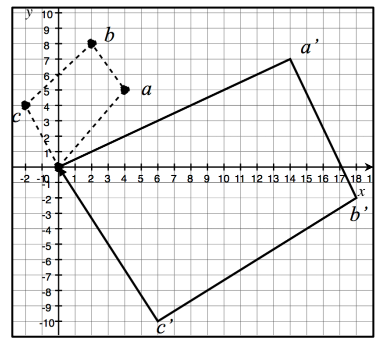
We can do this with as many vectors as we wish but all the lines on the graph will become cumbersome. We will use 3 vectors this time and rather than having 3 separate 1×2 matrices, we will use one 3×2 matrix. Also instead of connecting them to the origin, let's connect them to the previous one and return to the origin. In a sense then, we are creating a geometric object.



$$A=\begin{bmatrix}4\\5\end{bmatrix}, B=\begin{bmatrix}2\\8\end{bmatrix}, C=\begin{bmatrix}-2\\4\end{bmatrix} \text{ or } \begin{bmatrix}4 & 2 & -2\\5 & 8 & 4\end{bmatrix}$$ as shown to the right.

Let's modify our vectors with the modification matrix $M=\begin{bmatrix}1 & 2\\3 & -1\end{bmatrix}$.

It can be shown that the absolute value of the determinant of the modification (or transformation) matrix M gives the magnitude of the dilation (the change in size) of the enclosed region under the transformation. The new shape is the same as the original one but rotated.

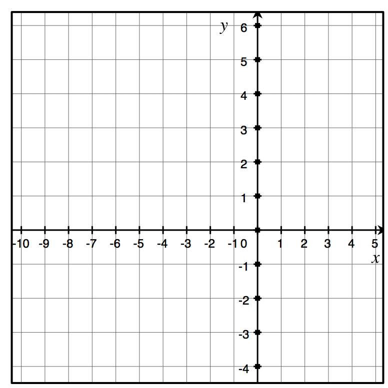


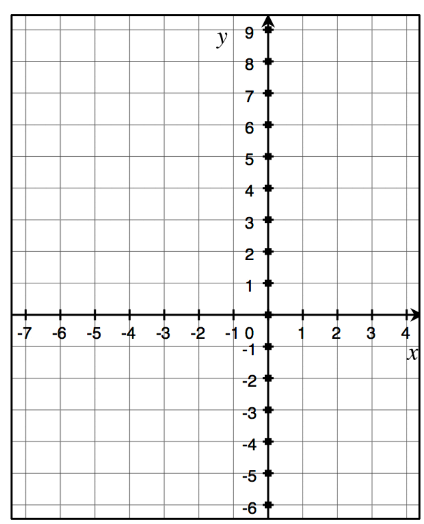
$$\begin{bmatrix}1 & 2\\3 & -1\end{bmatrix}\begin{bmatrix}4 & 2 & -2\\5 & 8 & 4\end{bmatrix}=\begin{bmatrix}14 & 18 & 6\\7 & -2 & -10\end{bmatrix}$$

$$\left\|\begin{matrix}1 & 2\\3 & -1\end{matrix}\right\|=|-1-6|=7$$ The new area is 7 times the original area.

Example 1) Given the vectors shown, find its linear transformation using the given modification matrix to create it and graph the result. Find the magnitude of the dilation.

a) $A=\begin{bmatrix}0\\-4\end{bmatrix}, B=\begin{bmatrix}-2\\6\end{bmatrix}, C=\begin{bmatrix}4\\4\end{bmatrix}, M=\begin{bmatrix}2 & -1\\1 & 0.5\end{bmatrix}$





b) $A=\begin{bmatrix}4\\0\end{bmatrix}, B=\begin{bmatrix}0\\2\end{bmatrix}, C=\begin{bmatrix}-4\\2\end{bmatrix}, D=\begin{bmatrix}-6\\0\end{bmatrix}, M=\begin{bmatrix}0.5 & -1\\-1.5 & -1\end{bmatrix}$

The *composition of two linear transformations* means that we apply a linear transformation to a set of vectors and immediately apply the second. It is a cumbersome process best done with a calculator.

Example 2) $A=\begin{bmatrix} 4 \\ 5 \end{bmatrix}, B=\begin{bmatrix} 2 \\ 8 \end{bmatrix}, C=\begin{bmatrix} -2 \\ 4 \end{bmatrix}$ and $M_1=\begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$ and $M_2=\begin{bmatrix} -4 & -3 \\ 1 & 2 \end{bmatrix}$.

a) Find the result of the two linear transformations. b) Show that the same result can be found by first taking the product $M_2 M_1$ of the transformation matrices and then applying it to the vectors.

If the result of a composition of linear transformations is the original vector matrix, then the two linear transformations are inverses to each other.

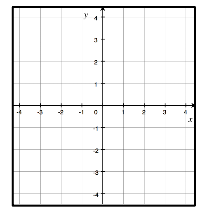
Example 3) Show in two ways that the composition of linear transformations with the following gives the original vector matrix.

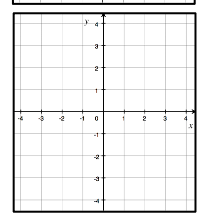
$$A=\begin{bmatrix} 4 \\ 0 \end{bmatrix}, B=\begin{bmatrix} 0 \\ 2 \end{bmatrix}, C=\begin{bmatrix} -4 \\ 2 \end{bmatrix}, D=\begin{bmatrix} -6 \\ 0 \end{bmatrix}, M_1=\begin{bmatrix} 2 & 4 \\ -4 & -10 \end{bmatrix}, M_2=\begin{bmatrix} 2.5 & 1 \\ -1 & -0.5 \end{bmatrix}$$

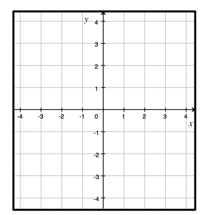
We have seen that the determinant of the modification (or transformation) matrix will dilate our original vectors. In a typical linear transformation that maps x to $mx + b$, the dilation is m. But there is another piece b to be determined. In the case of our linear transformation of vectors, we have the dilation but there is a rotation that we haven't accounted for. Here it is:

> The modification matrix associated with the linear transformation of a set of vectors is given by
> $M = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ where angle θ is measured counterclockwise from the original vector set.
> Since $|M| = \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} = |\cos^2\theta + \sin^2\theta| = 1$, the transformed vectors will have the same area as the original (dilation is 1).

Example 4) Suppose $A = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$, show the linear transformation for $\theta = 90°, 180°, 270°$.

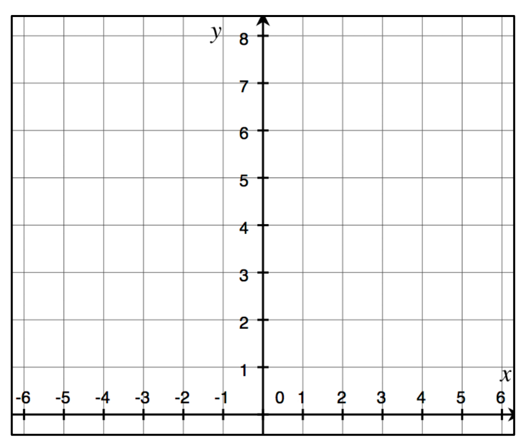






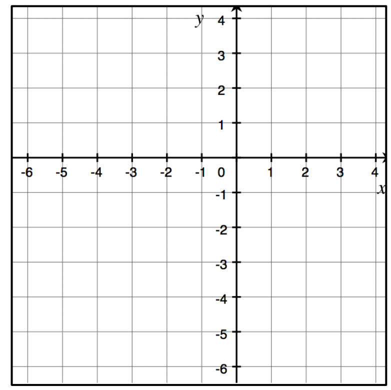
Example 5) $A = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 8 \end{bmatrix}$, $C = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$. a) Show the result of a $\theta = 60°$ linear transformation.

b) Show that the area of the original figure is the same as the transformed figure.



To double the size of the resulting area and rotate the matrices angle θ, $M = \begin{bmatrix} \sqrt{2}\cos\theta & -\sqrt{2}\sin\theta \\ \sqrt{2}\sin\theta & \sqrt{2}\cos\theta \end{bmatrix}$.

Example 6) $A = \begin{bmatrix} 4 \\ 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, C = \begin{bmatrix} -4 \\ 2 \end{bmatrix}, D = \begin{bmatrix} -6 \\ 0 \end{bmatrix}$, show the result of a $\theta = 45°$ linear transformation with a dilation of 2.

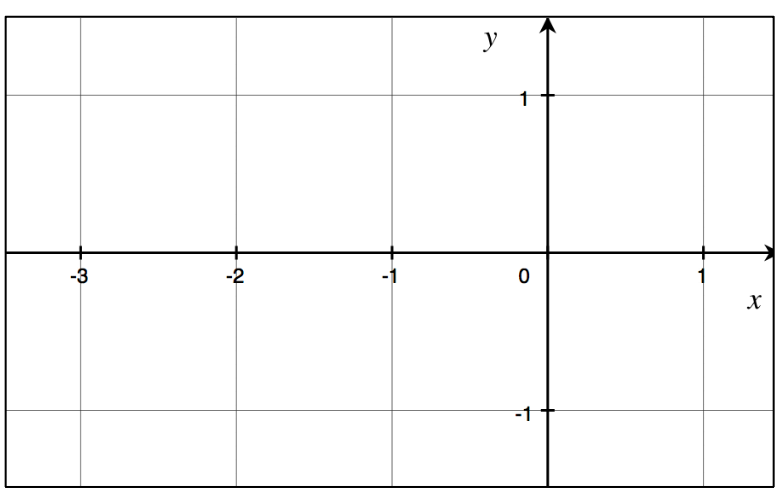


In general, to dilate the resulting area size by a factor of n and rotate the matrices angle θ counterclockwise, the transformation matrix is $M = \begin{bmatrix} \sqrt{n}\cos\theta & -\sqrt{n}\sin\theta \\ \sqrt{n}\sin\theta & \sqrt{n}\cos\theta \end{bmatrix}$.

Example 7) Use the unit square $A = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Perform the linear transformations. Graph the original figure and the figures created by the transformation below on the same set of axes.

a) dilation 4, $\theta = 135°$

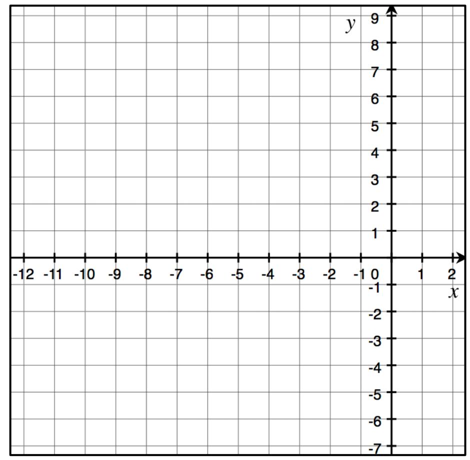
b) dilation 0.75, $\theta = 210°$



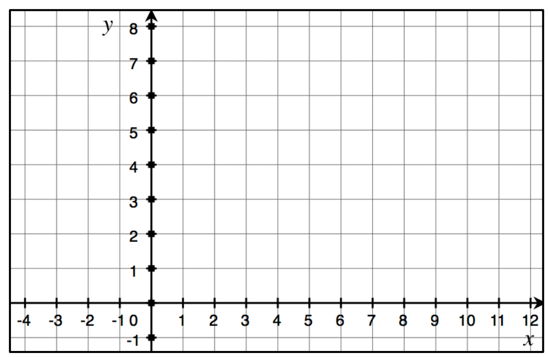
Topic 4.6 – Linear Transformations & Matrices – Homework

1. Given the vectors shown, find its linear transformation using the given modification matrix to create it and graph the result. Also find the magnitude of the dilation.

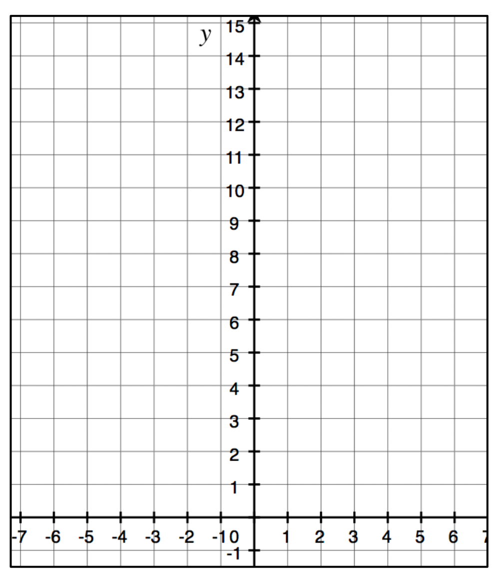
a. $A=\begin{bmatrix}2\\3\end{bmatrix}, B=\begin{bmatrix}-4\\8\end{bmatrix}, M=\begin{bmatrix}-1 & -2\\0.5 & -0.5\end{bmatrix}$



b. $A=\begin{bmatrix}2\\2\end{bmatrix}, B=\begin{bmatrix}0\\4\end{bmatrix}, C=\begin{bmatrix}-4\\3\end{bmatrix}, M=\begin{bmatrix}-1.5 & 2\\1 & 1.5\end{bmatrix}$



c. $A=\begin{bmatrix}0\\4\end{bmatrix}, B=\begin{bmatrix}2\\7\end{bmatrix}, C=\begin{bmatrix}6\\5\end{bmatrix}, D=\begin{bmatrix}4\\-1\end{bmatrix}, M=\begin{bmatrix}0 & -1\\0.5 & 2\end{bmatrix}$



2. $A=\begin{bmatrix} -8 \\ 2 \end{bmatrix}$, $B=\begin{bmatrix} 3 \\ -4 \end{bmatrix}$, $C=\begin{bmatrix} -5 \\ 1 \end{bmatrix}$ and $M_1=\begin{bmatrix} -6 & -2 \\ 2 & -4 \end{bmatrix}$ and $M_2=\begin{bmatrix} 5 & -1 \\ 0 & -3 \end{bmatrix}$.

a. Find the result of the two linear transformations. b. Show that the same result can be found by first taking the product M_2M_1 of the transformation matrices and then applying it to the vectors.

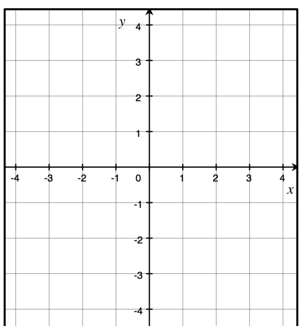
3. Show in 2 ways that the composition of linear transformations with A gives the original vector matrix.

$$A=\begin{bmatrix} 5 \\ 1 \end{bmatrix}, B=\begin{bmatrix} 2 \\ 6 \end{bmatrix}, C=\begin{bmatrix} -3 \\ 3 \end{bmatrix}, D=\begin{bmatrix} -7 \\ -2 \end{bmatrix}, M_1=\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, M_2=\begin{bmatrix} -2 & 1 \\ 1.5 & -0.5 \end{bmatrix}$$

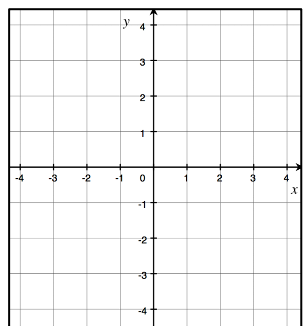
4. If vectors are linearly transformed by the following modification matrices, what is the resulting dilation and explain its meaning.

a. $\begin{bmatrix} -7 & -6 \\ 2 & 3 \end{bmatrix}$ b. $\begin{bmatrix} -3/2 & 3/2 \\ -1/2 & 4/9 \end{bmatrix}$ c. $\begin{bmatrix} -1/2 & 1/4 \\ 5/4 & -5/8 \end{bmatrix}$

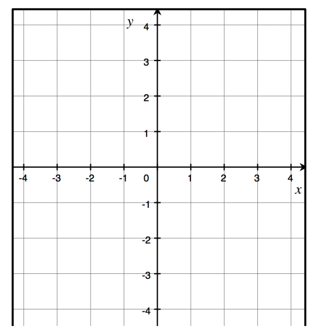
5. Suppose $A=\begin{bmatrix} 2 \\ 0 \end{bmatrix}, B=\begin{bmatrix} 4 \\ 2 \end{bmatrix}, C=\begin{bmatrix} 4 \\ -1 \end{bmatrix}$, show the linear transformation for $\theta = 90°, 180°, 270°$.



$\theta = 90°$



$\theta = 180°$



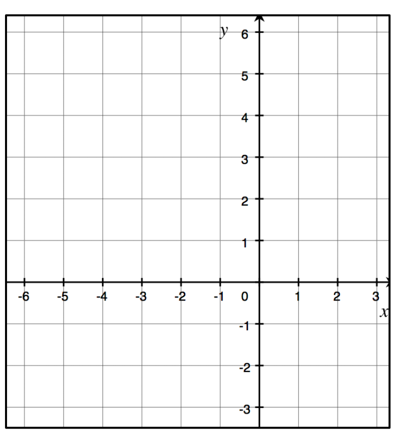
$\theta = 270°$

6. Use $A=\begin{bmatrix} 3 \\ 3 \end{bmatrix}, B=\begin{bmatrix} 0 \\ 4 \end{bmatrix}, C=\begin{bmatrix} -3 \\ 3 \end{bmatrix}$. a. Perform the following linear transformations.

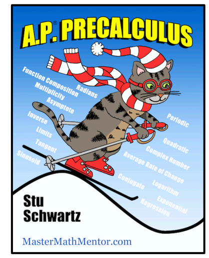
b. Graph the original figure and the figures created by the transformation below on the same set of axes.

a) dilation ¼ , $\theta = 315°$

b) dilation 2.25, $\theta = 30°$



Topic 4.7 – Matrix Applications – Classwork



Finally, we will examine a concept that utilizes a mathematical model that combines probability and matrices to analyze what is called a *stochastic process*, which consists of a sequence of trials satisfying certain conditions. The sequence of trials is called a *Markov Chain,* named after a Russian mathematician called Andrei Markov (1856-1922) .

A Markov chain is a mathematical system that experiences transitions from one state to another according to certain probability rules. The defining characteristic of a Markov chain is that no matter how the processs arrived at its present state, the possible future states are fixed. In other words, the probability of transitioning to any particular state is dependent solely on the current state and time elapsed. The *state space*, or set of all possible states, can be anything: letters, numbers, weather conditions, baseball scores, or stock performances.

For instance, there could be two states: raining or not raining, sleeping or awake, using electricity or not, hungry or full, having a pet or not. We are either in one state or another and move from state to state over time.

There could be 3 states: walking, running, or stationary, a person goes from happy to sad to neither, a family uses cable TV, streaming, or neither. We can move from any state to any state over time.

We start with a square *transition probability matrix P*. For simplicity purposes, let's make it a 2×2 matrix, meaning that there are two possible states, A and A'. The probability of moving from state A to state A' is r. The probability of moving from state A' to state A is s. Since there are only two possible states, the probability of staying in state A is $1 - r$. The probability of staying in state A' is $1 - s$. This can be summarized in matrix P to the right. Note that in either state, you either stay there or move to the other state so the row probabilities sum to one.

Next State

Current State

$$\begin{array}{c} \\ A \\ A' \end{array}\begin{array}{c} \begin{array}{cc} A & A' \end{array} \\ \begin{bmatrix} 1-r & r \\ 1-s & s \end{bmatrix} \end{array} = P$$

We also have an *initial state distribution matrix*. This is the set of initial probabilities (or percentages) of being in state A and state A'. Since there are only two states, the row proabilities sum to one.

$$\begin{array}{c} \quad\; A \qquad A' \\ S_0 = \begin{bmatrix} t & 1-t \end{bmatrix} \end{array}$$

If we multiply the 1×2 initial state matrix by the 2×2 transition probability matrix, we obtain the first state 1×2 matrix. If we multiply the first state matrix by the transition matrix, we obtain the second state matrix. If this process is repeated, we get the the kth state matrix.

$$S_1 = S_0P$$
$$S_2 = S_1P$$
$$\ldots$$
$$S_k = S_{k-1}P$$

If we multiply the the 1×2 initial state matrix by the inverse of 2×2 transition probability matrix, we obtain the state that is one iteration prior to the initial state.

Example 1) Suppose the only Italian restaurant in a small town is a Pizza Hut. So if people in that town want to eat out in an Italian restaurant, they must go to Pizza Hut. One day, a new Italian restaurant called the Raven opens and people who want to eat Italian now have a choice. A few people try it in the first week.

a) Suppose the initial state matrix and transition probability matrix (for one week later) is given below. Interpret it.

b) Examine and interpret the the probability that customers who go out to eat Italian in this town will eat at Pizza Hut and the Raven over the first 4 weeks.

It appears that as time goes on, the chances are greater that someone who wants to eat Italian in this town will go to the Raven. Will Pizza Hut survive? Since for each state, we are multiplying our result by our transition probability matrix, realize that $S_k = S_{k-1} \cdot P^k$.

c) Examine the 5th, 10th, 20th, and 30th state. What seems to be happening?

When the state matrix as the number of iterations gets larger and larger doesn't change, it is said to reach a limiting matrix and this matrix is called a *stationary matrix***.** So, if nothing changes, the probability that someone who wants to eat Italian in this town will go to Pizza Hut is 28.6% and the Raven is 71.4%.

d) When a stationary matrix is reached, each row of P^k is the stationary matrix. Show this occurs with S_{30}.

Not all initial state matrices transition into a stationary matrix. It is also possible that no one ends up at Pizza Hut. Take a course in linear algebra to further learn about how to model these situations.

Example 2) Currently, for people who drive cars, 69% of them are sedans and 31% are SUV's. 65.9% of sedan drivers are driving a sedan next year while 34.1% are driving SUV's. 66.2% of SUV drivers are driving a SUV next year while 33.8% are driving sedans. It is said that eventually, more than 50% of car drivers will be driving an SUV. Determine whether this is true and if so, how long it will take to occur?

Example 3) In a retirement home, 70% of the residents use a landline only, 20% use a cellphone only, and 10% use both. Every 6 months, the probabilities that residents use landlines, cellphones, and both change according to the following: For those who use landlines only, 55% are likely to continue using landlines only, 15% are likely to change to cellphone only, and the rest are likely to use both. For those who use cellphones only, 65 % are likely to continue using cellphones only, 5% are likely to change to a landline only, and the rest are likely to use both. For those who use both, 45% are likely to continue to use both, 5% are likely to change to a landline only, and the rest are likely to change to a cellphone only.

a) Create the initial state matrix and the transition probability matrix.

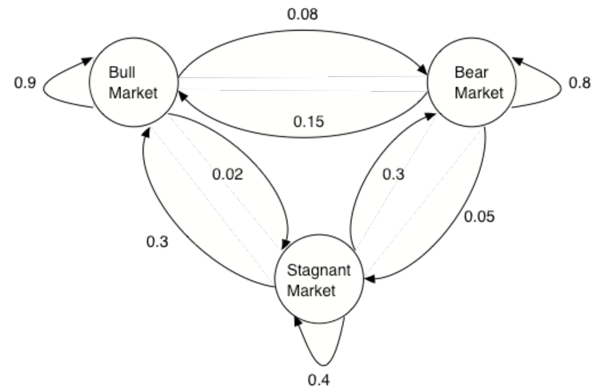
b) Calculate 6 iterations of change and interpret the last one.

c) Using your calculator set to 3 decimal places, about how many years will it take for the state matrix to become stationary. When that happens, what will it be?

Markov chains can help in predicting the behavior of the stock market over time. For the stock market, we define the following conditions over a period of time:

- bull market: where prices are generally rising due to investors having optimistic hopes for the future.
- bear market: where prices are generally declining due to investors having pessimstic hopes for the future.
- stagnant market: where the market is neither rising nor falling in general prices.

Example 4) Consider a hypothetical stock market with Markov properties that has the properties of the figure to the right over a period of a week.



a) Generate the probability transition matrix *P*.

b) Generate the initial state for markets that are completely bullish, completely bearish, and completely stagnant.

c) From the completely bullish market, examine the state of the market for each week of the month.

d) From the completely bullish market, examine the state of the market for each quarter of the current year.

e) From the completely bearish market, examine the state of the market for each quarter of the current year.

f) From a completely stagnant market, examine the state of the market for each quarter of the current year.

g. What do the results of d, e, and f seem to indicate?

h. Suppose the market currently has equal probability of being bullish, bearish, or stagnant. What was the previous state?

Markov chains show up in interesting and unexpected ways. Mobile devices have had predictive typing for a long time now, but whether you are using Android or IOS, there is a good chance that your typing app uses Markov chains. As you type, your words are analyzed and incorporated into the app's Markov chain probabilities. That is why your writing app presents 3 or more options, in order of most to least probability. It cannot know what you mean to type next but it is amazing how often it is correct.

When you do a Google search, each website is a state and the links between websites are transitions with probabilities. No matter which webpage you start on, your chance of landing on a certain webpage *X* is a fixed probability, assuming an infinitely long time of surfing. That is the basis of how Google ranks webpages. The higher the "fixed probability" of arriving at a certain webpage, the higher its PageRank. This is because a higher fixed probability implies that the webpage has a lot of incoming links from other webpages — and Google assumes that if a webpage has a lot of incoming links, then it must be valuable. The more incoming links, the more valuable or popular it is.

Example 5) Suppose that a Google search on an obscure topic yields only 4 "hits": pages A, B, C, and D, each containing links to the other pages. When presented with those 4 choices, the first table below gives the percentage of people who first go to that page.

The second table gives the percentage of people who link to the other pages. If a page links to itself, it means that the person stays at that page and does not link to another page. That should be the goal of a good website – someone goes to it and has no desire to check out another site.

A	B	C	D
80%	15%	5%	0%

	A	B	C	D
A	10%	25%	36%	29%
B	50%	10%	0%	40%
C	0%	15%	50%	35%
D	20%	40%	10%	30%

a) If the user only looks at one page, explain why the percentages are most likely what they are?

b) If the user has the option of looking at 2 pages, rank the pages in terms of popularity. Explain the results.

c) If the user can switch again, rank the pages. in terms of popularity. Explain why this happened.

Topic 4.7 – Matrix Applications – Homework

1. In Miami during the summer, if it is a clear day today, the probability it will be clear tomorrow is 90% and the probability it will rain tomorrow is 10%. If it is a rainy day today, the probability it will rain tomorrow is 25% and the probability it will be clear is 75%.

   In Seattle during the summer, if it is a clear day today, the probability it will be clear tomorrow is 40% and the probability it will rain tomorrow is 60%. If it is a rainy day today, the probability it will rain tomorrow is 80% and the probability it will be clear is 20%.

   In both cities, suppose that the probability it will be clear today is 50%. Give a 4-day forecast for each city.

   Suppose the probability it is clear today in Miami is 80% and the probability it is clear today in Seattle is 20%. What most likely was the weather in both cities yesterday?

2. At a restaurant, its dishes are either stacked and waiting for use, in use, or being washed. Following is their projected use 15 minutes from any time:

   • If stacked, they have a 72% chance of being stacked and a 26% chance of being in use in 15 minutes.
   • If in use, they have an 21% chance of being stacked and a 55% chance of being in use in 15 minutes.
   • If being washed, they have an 85% chance of being stacked and a 14% chance of being in use in 15 min.

   a. Create the probability transition matrix.

b. If at 7:00 PM, 80% of the dishes are stacked and 16% are in use, describe the state of the dishes over the next hour.

c. When the restarant opens at 5 PM, all the dishes are stacked. Describe their state over the next hour.

3. On a cruise ship, there are a number of dryers that are use for clothes and towels. There are 4 states of the machines: I: Idol and awaiting work W: Working on a job B: Broken R: In repair

The transition matrix T to the next state hourly is:

$$\begin{array}{c} \\ I \\ W \\ B \\ R \end{array} \begin{array}{c} \begin{array}{cccc} I & W & B & R \end{array} \\ \begin{bmatrix} 0.05 & 0.93 & 0.02 & 0 \\ 0.10 & 0.86 & 0.04 & 0 \\ 0 & 0 & 0.8 & 0.2 \\ 0.5 & 0.1 & 0 & 0.4 \end{bmatrix} \end{array}$$

a. At 6 AM, all dryers are in working condition but not in use. What is the state at 7 AM?

b. Find the state of the dryers every three hours through midnight.

c. At 12 AM, there is little need for drying and most of the machines get turned off and broken ones are repaired for the next day. But if this didn't happen, what, if any, is the final state of the dryers?

4. Stan is undergoing therapy. He finds he has severe mood swings based on the time of the week, going from happy to depressed to neither. His psychiatrist has asked him to keep a diary of his moods over a long period and the probability that he moves from state to state can be summarized below. The psychriatrist finds that the probabilities change based on the time of the week.

From Sunday night to Thursday night

	Happy	Neither	Depressed
Happy	0.22	0.38	0.40
Neither	0.08	0.34	0.58
Depressed	0.02	0.21	0.77

From Thursday night to Sunday night

	Happy	Neither	Depressed
Happy	0.62	0.28	0.10
Neither	0.24	0.47	0.29
Depressed	0.69	0.16	0.31

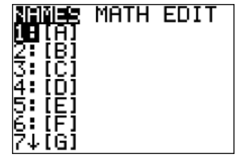
a. Create two matrices (D for weekdays and E for weekend) that summarize the above.

b. June 1 is a Saturday night and Stan has a 25% chance of being happy and a 25% chance of being depressed. Track his emotional state over the next three weeks. (use the morning).

c. Show Stan's two emotional states depending on the day of the week over 12 weeks.

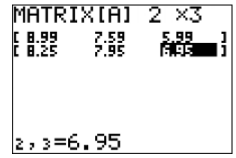
d. Show that Stan's two emotional states are not dependent on his initial mood state by having him start completely happy or completely depressed.

Using the Calculator:

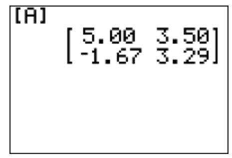


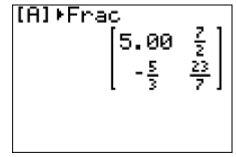
Any matrix operations are located on the 2[nd] MATRIX menu. Pressing it gives you the screen to the right.

Entering a Matrix:



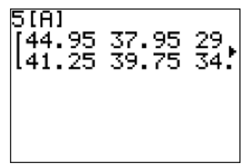
To enter a matrix, you first must select a matrix from the list and use EDIT. You must then decide on the matrix size. Inputting our Pizza/Wing cost matrix, we choose a 2 by 3 matrix and using the arrow buttons, we input our matrix.

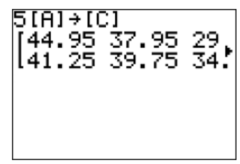




Displaying a Matrix: A matrix is displayed by using the 2[nd] MATRIX menu and choosing the matrix you want. All elements will be accurate to the number of decimal places set up in the MODE menu. You can also change matrices to display in fraction form by using the Math 1:->Frac command.

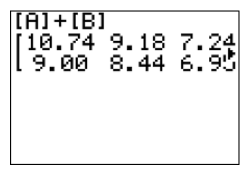
Scalar Multiplication





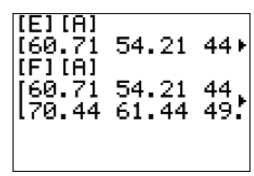
We can do scalar multiplication off the Home screen as shown. Use the arrow right button to scroll through the entire matrix. If we want to store the matrix as *C*, we can do so

Matrix Addition



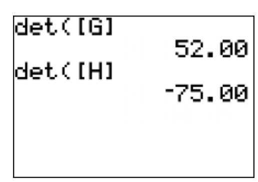
You can add matrices of the same dimensions. Here we add our price list for pizza and wings to the price list for toppings and dips. If we attempt to add two matrices with different dimensions, we will get a DIM MISMATCH error.

Matrix Multiplication



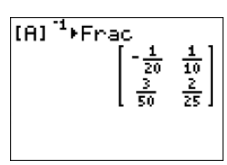
You can multiply two matrices as long as they are in the form of $(m \times n)$ multiplied by $(n \times r)$. Multiplying our (1×2) number matrix by our (2×3) cost matrix is shown here. As well as our (2×2) number matrix by our (2×3) cost matrix.

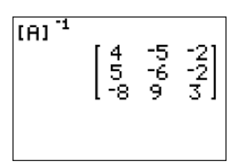
Determinants



You can find the determinant of any square matrix. The determinant statement is found in the 2[nd] MATRIX MATH 1:det(menu. The determinants for the matrices in classwork example # 8a) and # 9a) are shown.

Inverses





To find the inverse of a matrix, input it, display it on the home screen and then use the x^{-1} key.