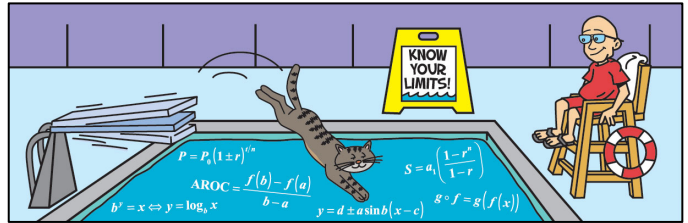


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3.1) Radian/Degree Measurement (Calculator)

1. Arrange these angles from highest to lowest in value.

- | | | | |
|-------------------|-------------------|-------------------|-------------------------------|
| I. 155° | II. $(8\pi/9)^R$ | III. 0.44 rev | IV. $0.5 \text{ rev} - 0.5^R$ |
| A) IV, III, II, I | B) III, I, II, IV | C) I, III, IV, II | D) II, III, I, IV |

2. One angle of a triangle is 88.25° while another is $21^\circ 18' 45''$. Find the 3rd angle.

- | | | | |
|------------------------|------------------------|------------------------|------------------------|
| A) $70^\circ 16' 15''$ | B) $70^\circ 26' 15''$ | C) $71^\circ 16' 15''$ | D) $71^\circ 26' 15''$ |
|------------------------|------------------------|------------------------|------------------------|

3. Odessa, Ukraine, which lies north of the equator, lies on the same longitude as South Durban, South Africa which lies at $29^\circ 53'$ South. If the flying distance between the two cities is 5,276 miles and the radius of the earth is 3,960 miles, what is the latitude of Odessa?

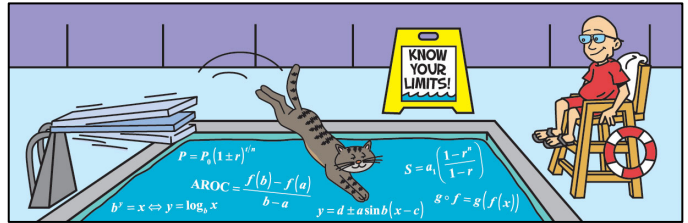
- | | | | |
|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| A) $46^\circ 48' \text{ N}$ | B) $46^\circ 27' \text{ N}$ | C) $76^\circ 34' \text{ N}$ | D) $76^\circ 20' \text{ N}$ |
|-----------------------------|-----------------------------|-----------------------------|-----------------------------|

4. The earth rotates about its axis once every 23 hours, 56 minutes, 4 seconds. If the radius of the earth is 3,960 miles, find the linear velocity of a point on the equator.

- | | | | |
|---------------|---------------|---------------|----------------|
| A) 165.45 mph | B) 168.08 mph | C) 1039.6 mph | D) 2067.07 mph |
|---------------|---------------|---------------|----------------|

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3.2) The Trig Functions (No Calculator)

- If $P(\sqrt{6}, \sqrt{10})$ is a point on the terminal side of θ , what is the value of $\sin\theta \cos\theta \tan\theta$?

A) $\frac{5}{2}$ B) $\frac{3}{2}$ C) $\frac{3}{8}$ D) $\frac{5}{8}$

- α is an angle in which $\csc\alpha$ and $\sec\alpha$ are both positive. β is an angle in which $\csc\beta$ and $\sec\beta$ are both negative. In what quadrant is $\beta - \alpha$?

A) I B) II C) III D) I or II

- Which of the following are undefined?

I. $\sin 0^\circ + \csc 0^\circ$ II. $\cos 180^\circ + \sec 180^\circ$ III. $\tan 270^\circ + \cot 270^\circ$

A) I and II B) I and III C) II and III D) I, II and III

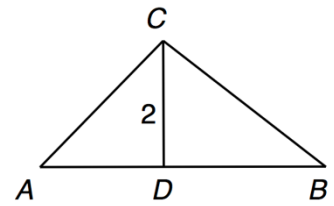
- Find difference between the largest and smallest value of:

I. $-3\tan^2 210^\circ + 6\cot^2 120^\circ$ II. $(5\sin 330^\circ - \cos 240^\circ)^2$ III. $\frac{-5\cos 180^\circ - 8\cos 300^\circ}{-5\cos 180^\circ + 8\cos 300^\circ}$

A) 5 B) $\frac{35}{9}$ C) 4 D) 3

3.2) The Trigonometric Functions

5. Triangle ABC , drawn to scale, is shown to the right with $\angle C = 105^\circ$ with $\angle ACD$ and $\angle BCD$ special angles.



a. Find $\angle ACD$ and $\angle BCD$. **(1)**

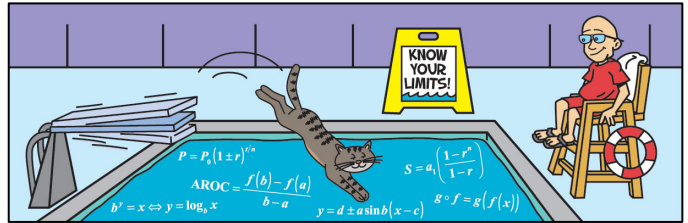
b. Find the area of triangle ABC . **(3)**

c. If CD were doubled, how does that affect answer b? **(1)**

d. Find the perimeter of triangle ABC . **(3)**

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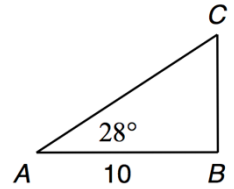
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3.3a) Using Trigonometric Functions (Calculator)

1. In triangle ABC shown to the right, $\angle A$ increases by 1° with AB staying constant. By how much does BC increase?

- A) 0.1534 B) 0.2260
C) 10 D) 0.0833



2. Find $\cot^{-1}(0.8887)$

- A) $48^\circ 22' 21''$ B) $48^\circ 37' 25''$ C) $0^\circ 2' 40''$ D) $0^\circ 1' 27''$

3. A right triangle ABC has a known hypotenuse c as well as one other side b . Which trig function could be used to find A before any other part of the triangle?

I. sine II. cosine III. tangent

- A) I only B) II only C) Both I and II D) III only

4. What is the area of right triangle ABC with $A = 24^\circ$ and hypotenuse $c = 3$ ft, 6in ?

- A) 27.31ft^2 B) 4.55ft^2 C) 3.22ft^2 D) 2.28ft^2

3.3a) Using Trigonometric Functions

5. For each right triangle ABC, find the requested information: (sides accurate to 2 decimal places, Angles in degrees, minutes, seconds).

a. $A =$ $a =$
 $B = 38^{\circ}9'$ $b =$ Find the length of the longest leg. **(2)**
 $C = 90^{\circ}$ $c = 17.62$

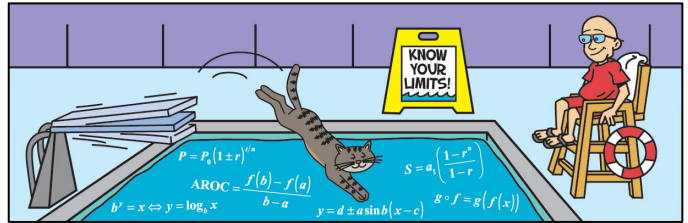
b. $A = 18.7^{\circ}$ $a = 1$
 $B =$ $b =$ Find the hypotenuse. **(2)**
 $C = 90^{\circ}$ $c =$

- c. Find the smallest angle of a 3-4-5 triangle. **(2)**

d. $A =$ $a = 1$ mile
 $B =$ $b = 500$ feet Find the smallest angle. **(2)**
 $C = 90^{\circ}$ $c =$

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3.3b) Using Trigonometric Functions (Calculator)

- The wind is such that for every 10 feet of string between George and the end of his kite, the kite rises 4.1 feet. What is the angle of elevation of the kite at any time during its flight?
 - $24^{\circ}12'$
 - $22^{\circ}18'$
 - $67^{\circ}42'$
 - $65^{\circ}48'$

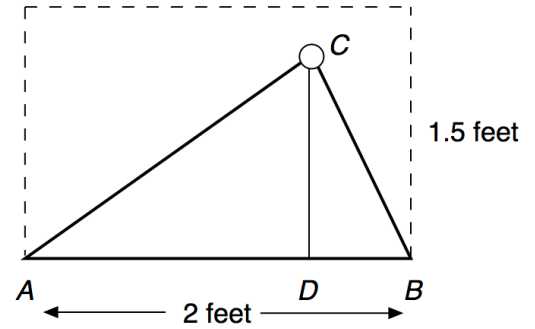
- The captain is on the bridge of a cruise ship, 200 feet above sea level as it is coming into port. He spots a light on the end of the dock at an angle of depression of 8° . The ship travels 400 feet closer to the dock. What is the angle of depression of the light?
 - 78.94°
 - 14.77°
 - 12.51°
 - 11.06°

- An airplane travels at 445 mph for 3.75 hours in a direction of 301° from Denver. What is its position relative to Denver?
 - 917 miles north
1,517 miles west
 - 917 miles west
1,517 miles north
 - 917 miles south
1,517 miles east
 - 917 miles east
1,517 miles south

- The heading of A from B is 45° . The heading of C from A is 135° . If $AB = 4$ km and $AC = 3$ km, what is the heading of B from C ?
 - 98.1°
 - 81.9°
 - 261.9°
 - not enough info

3.3b) Using Trigonometric Functions

5. Near the end of a 2-foot-wide plank of wood, a woodworker needs to draw a hole at the intersection of angle A and angle B as shown in the figure to the right. The plank is 1.5 feet long.



a. Write an expression for segment CD in terms of angle A . **(1)**

b. Write an expression for segment CD in terms of angle B . **(1)**

c. Write an expression for AD in terms of angles A and B . **(2)**

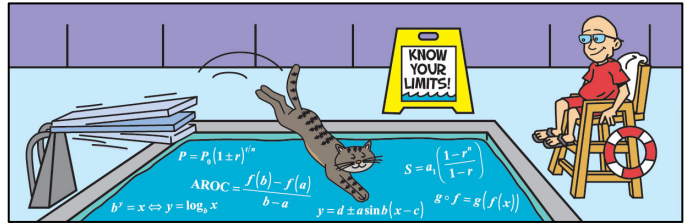
d. Using the answer from part c, write an expression for segment CD . **(1)**

e. If $A = 45^\circ$, write but do not solve, an inequality that compares the segment CD to the board length 1.5. **(1)**

f. Solve the inequality in e. to find the maximum value of angle B . **(2)**

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3.4a) Graphs of the Trig Functions (No Calculator)

1. For the graph of $f(x) = \frac{2 - 3\cos(30x + 60^\circ)}{2}$, which of the following describes the transformation on $y = \cos x$?

I. One unit up

II. Amplitude of 1.5

III. Period of 30°

IV. Shift of 60° left

A) I and II only

B) III and IV only

C) I, II and III only

D) I, II, III and IV

2. Three of the following have the same graph. Which is different?

A) $y = -\sin x$

B) $y = \sin(x - \pi)$

C) $y = -\cos\left(x + \frac{3\pi}{2}\right)$

D) $y = \cos\left(x - \frac{\pi}{2}\right)$

3. What is the period of $y = 4 - a \sin\left(\frac{2x}{a} - 3\right)$, $a \neq 0$?

A) $\frac{\pi}{a}$

B) πa

C) $\frac{2\pi}{a}$

D) $2\pi a$

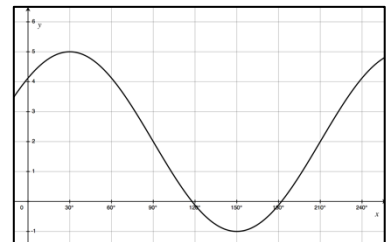
4. What is an equation of the graph to the right?

A) $y = 2 + 3\cos\left(\frac{3}{2}x - 30^\circ\right)$

B) $y = 2 + 3\cos\left(\frac{2}{3}x - 30^\circ\right)$

C) $y = 2 + 3\cos\left(\frac{3}{2}x - 45^\circ\right)$

D) $y = 2 + 3\cos\left(\frac{2}{3}x - 45^\circ\right)$

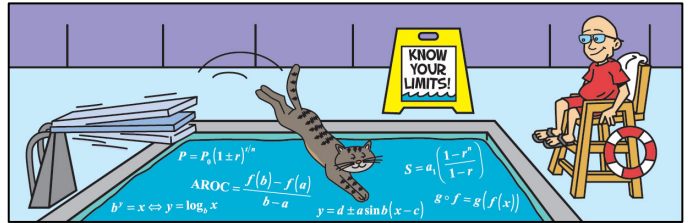


3.4a) Graphs of the Trig Functions

5. A trig function hits its minimum value of -15.6 at $x = -10.1$. It hits its maximum value of -1.2 at $x = 14.9$ but it also hits this maximum value at two other intermediary x -values to the left of $x = 14.9$.
- What is the period of the function? Explain your analysis. **(1)**
 - Give the two-intermediary x -values where the function also hits its maximum? **(2)**
 - Find the vertical translation and the amplitude of the function. **(2)**
 - Write the function in the form $y = d + a \cos b(x - c)$ where c is the smallest possible positive value **(2)**
 - If this function were to be written in the form $y = d + a \sin b(x - c)$, where c is the smallest possible positive value, what is c ? **(1)**

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3.4b) Graphs of the Trig Functions (Calculator)

- A trig function in the form $y = d + a \sin b(x - c)$ has vertical translation 1, amplitude 0.5, period 120° , and phase shift 5° to the left. What is the y -intercept?

A) 0.6749 B) 0.8706 C) 1.1294 D) not enough information
- Let $f(x) = \sin x$. Let $g(x)$ be the transformation of f with double the amplitude and double the period. Let $h(x)$ be the transformation of f with half the amplitude and half the period. Find the number of intersections of g and h on $[0, 360^\circ)$.

A) 1 B) 2 C) 4 D) 5
- Find the value of $\tan(\sec^{-1}(-4)) + \sec(\tan^{-1}(-4))$

A) -7.996 B) 0.250 C) -2.842 D) 7.996
- Suppose a trig function f passes through these points. What is the first positive zero of f ?

x	-8.8	-6.2	-3.6	-1.0	1.6
y	-9.3	-3.9	1.5	-3.9	-9.3

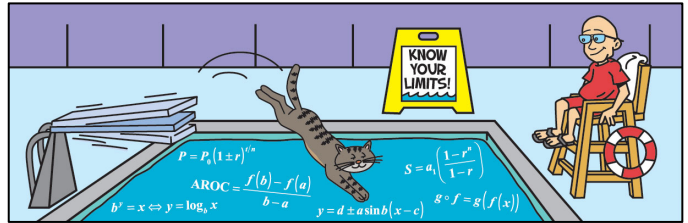
A) 0.164 B) 2.236 C) 2.294 D) 5.536

3.4b) Graphs of the Trig Functions

5. John is bungee jumping off a bridge that is 130 feet high. His low point is over water 10 feet high which he reaches in $k = 3$ seconds.
- a. Assuming John then settles into simple harmonic motion during his fall, find the equation that describes his height above water as a function of t . **(3)**
- b. After John reaches his lowest point, he doesn't come back to bridge level and doesn't fall back quite as low. If his harmonic height in part a) is in the form $h(t) = d + a \cos bt$, his true height is given by $H(t) = d + (a \cos bt)e^{-0.5(t-k)}$ where k is the time he hits his lowest height. Write the equation and determine his height in 20 seconds and the direction he is moving. **(3)**
- c. Looking at the graph, determine the two times between which his fall is first less than 50 feet. **(1)**
- d. Suppose the bungee company allows John to pretty much come to a stop before hauling him back up to the bridge. What is the distance he gets hauled up? Explain your analysis. **(1)**

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3.5) Analytical Trig (No Calculator)

1. Which of the following expressions are equal to 1?

I. $(1 + \tan^2 x)(1 - \sin^2 x)$ II. $\frac{\tan^2 \theta}{\sin^2 \theta} - \tan^2 \theta$ III. $\cot x - \csc x(\cos x + \sin x)$

- A) I and II only B) I and III only C) II and III only D) I, II and III

2. Find the exact value of $\tan(345^\circ)$

A) $\frac{-\sqrt{3}}{3}$ B) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ C) $\frac{\sqrt{3}-2}{\sqrt{3}+2}$ D) $\frac{\sqrt{3}-3}{\sqrt{3}+3}$

3. Given $\sin A = \frac{3}{5}$, A in quadrant II, and $\tan B = 2$, B in quadrant III, find $\cos(2A - B)$

A) $\frac{-4\sqrt{5}}{25}$ B) $\frac{4\sqrt{5}}{5}$ C) $\frac{-11\sqrt{5}}{25}$ D) $\frac{41\sqrt{5}}{125}$

4. Which of the following is equal to $\csc^2 12^\circ$?

A) $\frac{2}{1 + \cos 24^\circ}$ B) $\frac{2}{1 - \cos 24^\circ}$ C) $\frac{2}{1 + \cos 6^\circ}$ D) $\frac{2}{1 - \cos 6^\circ}$

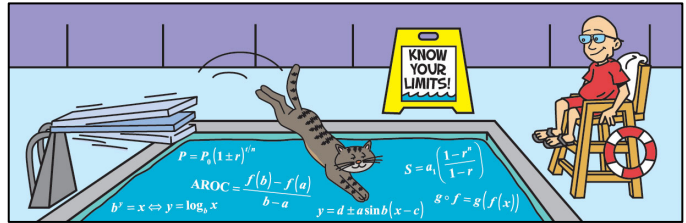
5. Solve the following equations.

a. $\cos x = \cos \frac{x}{2}$ for $[0^\circ, 360^\circ)$ (4)

b. $\tan 2x + \tan x = 0$ on $[0, \pi)$ (4)

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3.6a) Solving Oblique Triangles (Calculator)

1. 3 pieces of information are known about a potential triangle: 2 of its sides and one angle which is obtuse. How many possible solutions does the triangle have?

I: 0 Solutions

II: 1 Solution

III: 2 Solutions

A) II only

B) I and II only

C) I and III only

D) II or III only

2. In triangle ABC , $A = 47^\circ$ and $b = 31$. If neither of the remaining angles can be greater than 75° , which of the following is *not* a possible value for a ?

A) 23.49

B) 24.82

C) 26.29

D) 28.34

3. In triangle ABC , $A = 34^\circ$, $a = 36$ and $b = 64$. What are possible values of C ?

A) $C = 62.22^\circ$ only

B) $C = 49.78^\circ$ only

C) $C = 49.78^\circ$ or 62.22°

D) $49.78^\circ < C < 62.22^\circ$

4. In triangle ABC , find A if $a = 19$, $B = 40^\circ 30'$, and $c = 12$.

A) $25^\circ 28'$

B) $38^\circ 28'$

C) $101^\circ 13'$

D) $141^\circ 43'$

3.6a) Solving Oblique Triangles

5. A triangle has an angle of 38° and two sides of 29 and 46 but with no knowledge of the relationship of the sides to the angle. **(4)**

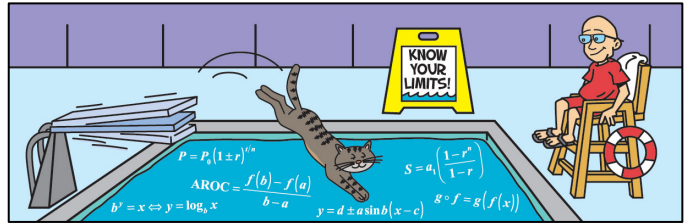
a. How many triangles fit the criteria? Justify your answer.

b. If neither of the known sides are opposite the known angle, determine the area of the triangle using only the given information. **(1)**

c. Using situation b) above, verify your answer using Heron's formula. **(3)**

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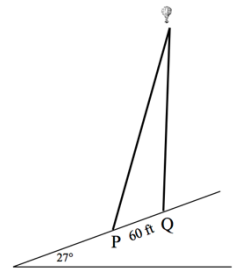
3.6b) Solving Oblique Triangles (Calculator)

1. Standing on the balcony of my 3rd floor apartment, I view the top of a palm tree at an angle of elevation of 12° and distance 24 feet and the base of the tree at an angle of depression of 61° and distance 53 feet. At what angle to the ground is the palm tree?

A) 63.48° B) 81.48° C) 87.52° D) 90°

2. Quincy is 60 feet ahead of Paul as they both climb a hill at an angle of elevation of 27° as shown in the figure to the right (not to scale). Paul views a hot air balloon at an angle of elevation of 59° while Quincy views the balloon at an angle of elevation of 73° . What is the closest distance that either man is from the balloon?

A) 66.94 ft B) 131.43 ft C) 108.28 ft D) 178.41 ft

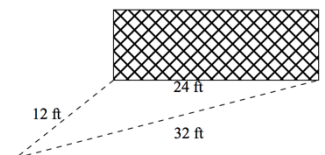


3. A ship is sailing due north. At a certain point, the captain notices a lighthouse 12.5 km away from the ship at a heading of 300.8° . The ship travels for 22 km. What is the heading of the lighthouse from the ship at that point in time?

A) 34.53° B) 145.47° C) 214.53° D) 235.47°

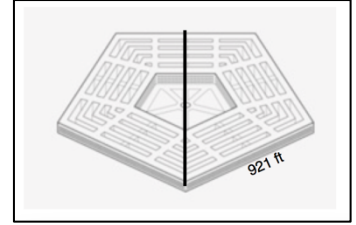
4. A soccer player has a wide-open net as shown in the figure to the right. Within what angle made by the ball and goalposts must he shoot to score?

A) 18.6° B) 39.6°
C) 42.4° D) 58.2°

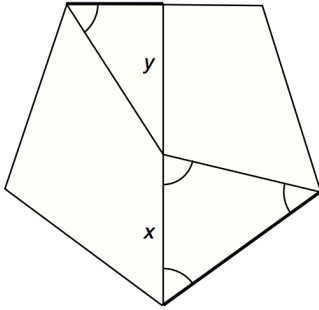


3.6b) Solving Oblique Triangles

5. The Pentagon in Washington has 5 outer walls, each measuring 921 feet. The walls create angles of 108° . Suppose a tunnel is to be built from the point to the midpoint of the opposite wall as shown in the figure.



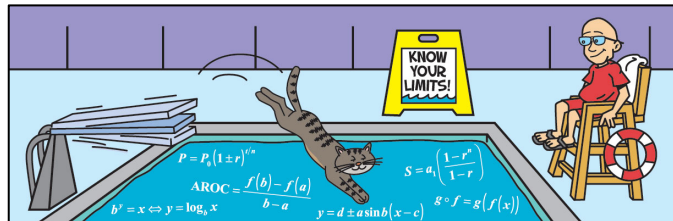
- a. On the diagram below, show the values of the known angles and sides. **(2)**



- b. What is the length of the tunnel x from the point to the center of the pentagon? Show your formula. **(1)**
- c. What is the length of the tunnel y from the center of the pentagon to the midpoint of the opposite side? **(1)**
Show your formula.
- d. How much shorter is walking through the tunnel than walking around the building? **(1)**
- e. Using the triangle with y , find the total footprint area of the pentagon. **(1)**
- f. Using the triangle with x and Heron's formula, verify part e) **(2)**

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3.7) Trig and Polar Coordinates (No Calculator)

1. Starting with the point $(-6 + 8i)$ and rotating it 90° clockwise gives which point?

- A) $(-8 + 6i)$ B) $(8 + 6i)$ C) $(-8 - 6i)$ D) $(6 - 8i)$

2. Write the complex number $z = -\frac{5\sqrt{6}}{2} + \frac{5i\sqrt{2}}{2}$ in trigonometric (polar) form.

- A) $10\sqrt{2}(\cos 300^\circ + i \sin 300^\circ)$ B) $10\sqrt{2}(\cos 120^\circ + i \sin 120^\circ)$
 C) $5\sqrt{2}(\cos 330^\circ + i \sin 330^\circ)$ D) $5\sqrt{2}(\cos 150^\circ + i \sin 150^\circ)$

3. If $z_1 = 8\sqrt{2}(\cos 325^\circ + i \sin 325^\circ)$ and $z_2 = 4(\cos 100^\circ + i \sin 200^\circ)$, find z_1/z_2

- A) $-2 - 2i$ B) $2\sqrt{2} + 2i\sqrt{2}$ C) $\sqrt{2}(\cos 3.5^\circ + i \sin 3.5^\circ)$ D) Calculator needed

4. Find $(\sqrt{0.5} - i\sqrt{0.5})^8$

- A) 1 B) i C) 8 D) $8i$

5. Let $z = -64i$

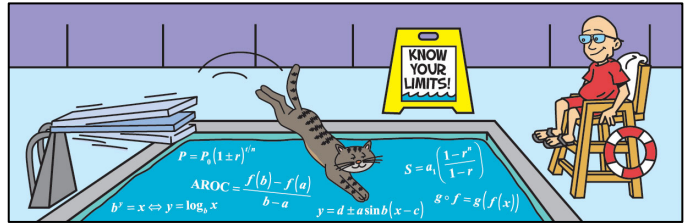
a. Write z in polar form. **(1)**

b. Find all three cube roots of z . **(6)**

c. For the root using the largest angle in part b), use Demoivre's Thm. to show that cubing it gives $-64i$. **(1)**

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3.8) Polar Graphs (No Calculator)

1. The graph of $r = \sin(\theta + 2)$ is the same as the graph of $r = \sin \theta$ except

- A) it is translated 2 units up
 B) it is translated 2 units to the left
 C) it is rotated about the origin
 D) it is the same

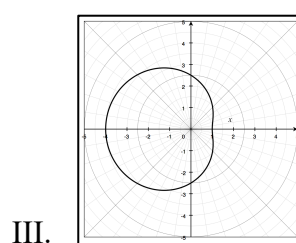
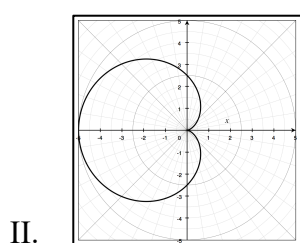
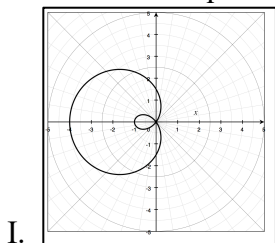
2. The graph of $r = \cos \theta - \sin \theta$ graphs

- A) a rose curve
 B) a cardioid
 C) a circle
 D) a parabola

3. The rate of change of $r = -1 - 3\cos \theta$ between $\theta = 240^\circ$ and $\theta = 270^\circ$, measured in units/radians is

- A) $\frac{-1}{20}$
 B) $\frac{-1}{20}$
 C) $\frac{-3}{\pi}$
 D) $\frac{-9}{\pi}$

4. Match the equation with the curve.



d. $r = 2.5 - 2.5 \cos \theta$

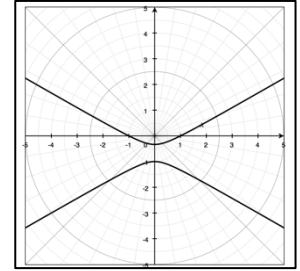
e. $r = 2.5 - 1.5 \cos \theta$

f. $r = 1.5 - 2.5 \cos \theta$

- A) I - f, II - d, III - e
 B) I - e, II - d, III - f
 C) I - e, II - f, III - d
 D) I - e, II - d, III - f

3.8) Polar Graphs

- 5a. Explain why the polar graph of $r = \frac{1}{1 - 2\sin\theta}$ matches the graph that is shown to the right. **(3)**



- 5b. Describe the shape of the following polar equations:

i. $r = 9\sin 3\theta$ **(2)**

ii. $r = 9\sin 6\theta$ **(1)**

iii. $r^2 = 9\sin 2\theta$ **(2)**