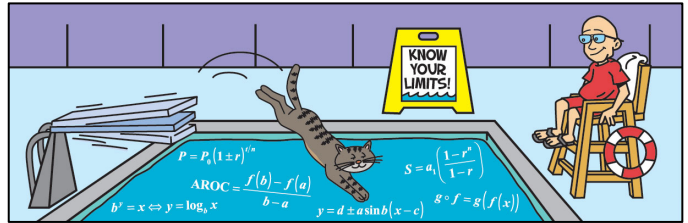


# Diving In

## AP Precalculus



### 4.1) Parametric Equations (No Calculator)

1. Change the parametric equations  $x = \sqrt{t-2}$  and  $y = t^2 - 4t$  to rectangular form stating any restriction.
- A)  $y = x^4 - 4, x \geq 2$       B)  $y = x^4 - 4, x \geq 0$       C)  $y = x^4 - 4x^2 - 4, x \geq 2$       D)  $y = x^4 - 4x^2 - 4, t \geq 2$

2. Change the circle with center  $C\left(-8, \frac{5}{2}\right)$  and radius  $\frac{5}{2}$  to parametric form.

A)  $x = \frac{5}{2} \cos t - 8$      $y = \frac{5}{2} \sin t + \frac{5}{2}$

B)  $x = \frac{5}{2} \cos t + 8$      $y = \frac{5}{2} \sin t - \frac{5}{2}$

C)  $x = \frac{25}{4} \cos t - 8$      $y = \frac{25}{4} \sin t + \frac{5}{2}$

D)  $x = \frac{25}{4} \cos t + 8$      $y = \frac{25}{4} \sin t - \frac{5}{2}$

3. If  $f(t) = \left(6 - \frac{t^2}{2}, 8 + 3t\right)$  find the slope of the parametric curve between  $t = -1$  and  $t = 3$ .

A)  $\frac{-24}{5}$

B) 5

C)  $\frac{-12}{5}$

D) -3

4. The graph of  $f(t) = (t^2 - 4, t^4 - t)$  has two  $x$ -intercepts and two  $y$ -intercepts. If  $X$  equals the distance between the  $x$ -intercepts and  $Y$  represents the distance between the  $y$ -intercepts, what is  $|X - Y|$  ?

A) 4

B) 3

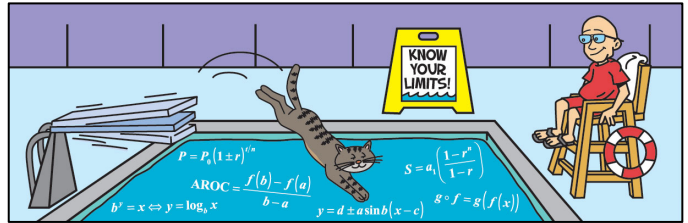
C) 1

D) 0



# Diving In

## AP Precalculus



### 4.2) The Conic Sections (Calculators Permitted)

1. Find the equation of the circle with center  $\left(-\frac{3}{2}, 2\right)$  passing through  $\left(\frac{1}{2}, -4\right)$

A)  $4x^2 + 4y^2 - 12x + 16y = 135$

B)  $4x^2 + 4y^2 + 12x - 16y = 135$

C)  $4x^2 + 4y^2 = 135$

D)  $4x^2 + 4y^2 - 6x + 16y = 135$

2. Find the directrix of the parabola  $y^2 + 10y + 12x = 11$

A)  $x = 0$

B)  $x = 6$

C)  $y = -2$

D)  $y = -8$

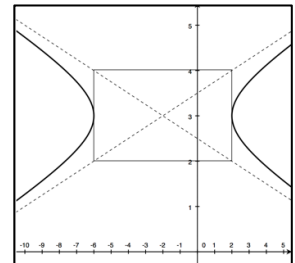
3. Find the equation of the conic to the right.

A)  $\frac{(x+2)^2}{16} - (y-3)^2 = 1$

B)  $(x+2)^2 - \frac{(y-3)^2}{16} = 1$

C)  $(y-3)^2 - \frac{(x+2)^2}{16} = 1$

D)  $\frac{(y-3)^2}{16} - (x+2)^2 = 1$



4. An ellipse with vertices  $(-1, -5)$  and  $(-1, 9)$  and eccentricity  $1/3$  is given by

A)  $\frac{(x+1)^2}{10} + (y-2)^2 = 1$

B)  $(x+1)^2 + \frac{(y-2)^2}{10} = 1$

C)  $\frac{(x+1)^2}{8} + (y-2)^2 = 1$

D)  $(x+1)^2 + \frac{(y-2)^2}{8} = 1$

5. Each of the problems below are concerned with conics.

Statuary Hall in the United States Capital is elliptical. It measures 46 feet wide and 96 feet long.

- a. Find a possible equation that describes the ellipse. **(1)**
  
  
  
  
  
  
  
  
  
  
- b. Lisa Tyree is standing at one focus and her husband Ted can hear her whisper, standing at the other foci. How far apart are they? Show how you get your answer. **(1)**
  
  
  
  
  
  
  
  
  
  
- c. Write the equation of the ellipse in parametric form. **(1)**

A car headlight has a parabolic cross-section with diameter of 8 inches and a depth of 6 inches.

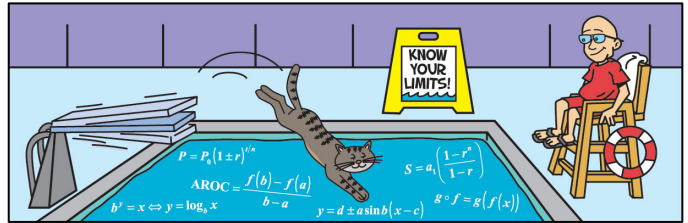
- d. Find an equation that describes the parabola. **(2)**
  
  
  
  
  
  
  
  
  
  
- e. The bulb is at the focus of the parabola. How far from the back of the headlight assembly is the bulb? **(1)**

An engineer determines that the equation of the structure he intends to build is  $x^2 - y^2 + 200y - 12500 = 0$ .

- f. Determine the only possible shape of the structure and determine and explain whether it could exist. **(2)**

# Diving In

## AP Precalculus



### 4.3) Vectors (Calculator Permitted)

1. Find the magnitude of the vector with initial point  $\left(\frac{-5}{2}, \frac{4}{3}\right)$  and terminal point  $\left(-2, \frac{-5}{6}\right)$ .

A)  $\frac{\sqrt{2}}{2}$

B)  $\frac{5\sqrt{2}}{2}$

C)  $\frac{\sqrt{178}}{6}$

D)  $\frac{\sqrt{610}}{6}$

2. Given the vectors  $\mathbf{u} = \langle -4, -3 \rangle$  and  $\mathbf{v} = \langle -1, -4 \rangle$ , find the unit vector for  $2\mathbf{u} - 4\mathbf{v}$ .

A) 1

B)  $\langle -4, 10 \rangle$

C)  $\left\langle \frac{-2}{\sqrt{21}}, \frac{5}{\sqrt{21}} \right\rangle$

D)  $\left\langle \frac{-2}{\sqrt{29}}, \frac{5}{\sqrt{29}} \right\rangle$

3. Find the angle between the vectors  $\mathbf{u} = \langle -8, 6 \rangle$  and  $\mathbf{v} = \left\langle 7, \frac{-1}{2} \right\rangle$

A)  $32.78^\circ$

B)  $90^\circ$

C)  $147.22^\circ$

D)  $168.50^\circ$

4. An object moves with velocity given by  $\mathbf{v}(t) = (\sin t)\mathbf{i} + (\sin t + \cos t)\mathbf{j}$ ,  $0 \leq t \leq \pi$ . Its speed must lie in which of the following intervals?

A) (1.017, 2.588)

B) (0.382, 2.618)

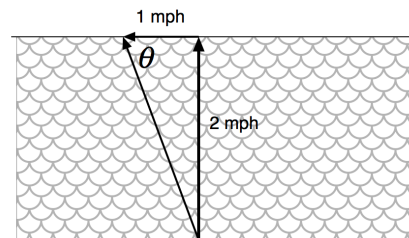
C) (0.618, 1.618)

D) (1, 2)

### 4.3) Vectors

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5. Swimmers at Kamp Kohut are swimming across Lake Thompson, a distance of 1 mile. They set out by swimming straight across the lake towards a red house but there is a cross current of 1 mph as shown in the figure to the right.

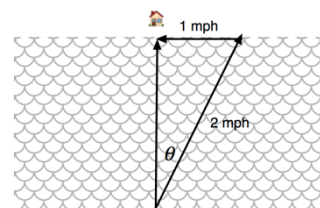


- a. Write a vector that describes the swimmers in still water and a vector representing the current.



- b. Write a vector that describes the true path of the swimmers due to the current and determine their speed.
- c. At what angle  $\theta$  will they hit the opposite bank?

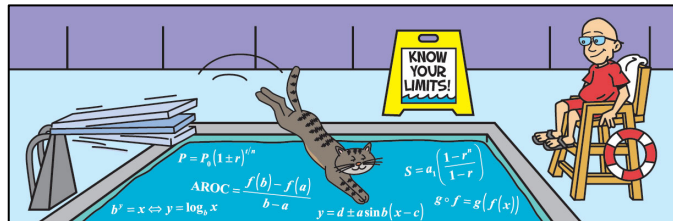
Ken is a stronger swimmer than the others. He figures that if he sets out at on angle  $\theta$  from part (c), swimming at 2 mph as shown in the figure to the right, it will bring him directly across the lake to the house.



- d. Write a vector that describes the Ken's position in still water.
- e. Show whether or not Ken is right in his thinking by writing a vector that describes Ken's true path. Determine his speed as well.

# Diving In

## AP Precalculus



### 4.4) Matrices (No Calculator)

1. If  $A = \begin{bmatrix} -6 & 2 \\ 4 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} -3 & -2 \\ -1 & 8 \end{bmatrix}$  find  $AB$

A)  $\begin{bmatrix} 18 & -4 \\ -4 & 8 \end{bmatrix}$

B)  $\begin{bmatrix} 16 & 28 \\ -13 & 0 \end{bmatrix}$

C)  $\begin{bmatrix} 10 & -8 \\ 38 & 6 \end{bmatrix}$

D)  $\begin{bmatrix} 20 & -16 \\ -11 & -16 \end{bmatrix}$

2. Find the value of  $\begin{vmatrix} 2\sqrt{48} & \frac{-5}{\sqrt{12}} \\ \frac{2}{\sqrt{3}} & 4\sqrt{3} \end{vmatrix}$

A)  $\frac{283}{3}$

B)  $-13$

C)  $3$

D)  $\frac{293}{3}$

3. For which of the following matrices will its inverse contain only integers?

I.  $\begin{bmatrix} 2 & -4 \\ -4 & 2 \end{bmatrix}$

II.  $\begin{bmatrix} 9 & -8 \\ 10 & -9 \end{bmatrix}$

III.  $\begin{bmatrix} \frac{3}{4} & -4 \\ 2 & 12 \end{bmatrix}$

A) I only

B) II only

C) III only

D) II and III only

4. Using matrices to solve the simultaneous equations:  $8x - 11y = m$   
 $-2x - 3y = n$  would give the result:

A)  $\begin{bmatrix} \frac{3}{2}m + \frac{11}{2}n \\ m + 4n \end{bmatrix}$

B)  $\begin{bmatrix} \frac{3}{2}m + \frac{11}{2}n & m + 4n \end{bmatrix}$

C)  $\begin{bmatrix} 8m - 11n \\ -2n + 3n \end{bmatrix}$

D)  $\begin{bmatrix} 8m - 11n & -2m + 3n \end{bmatrix}$

#### 4.4) Matrices

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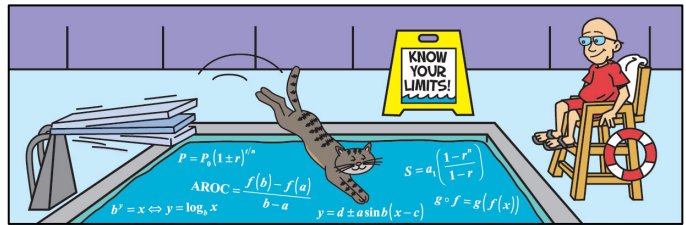
5. A supplier supplies Purina One cat food to 3 Pet shops locations in Pompano Beach in bags of 8 lb, 15 lb, and 32 lb. Each entry in the table shows the number of bags delivered and how many of them were sold in a week. The cost of each bag to the store is \$4, \$10, and \$15 respectively and the price that the store charges is \$10, \$20 and \$30 respectively.

	8 lb	15 lb	32 lb
Lyons Rd	$\frac{8}{6}$	$\frac{20}{15}$	$\frac{10}{2}$
Federal Hwy	$\frac{10}{10}$	$\frac{18}{8}$	$\frac{5}{0}$
Commercial Blvd	$\frac{12}{9}$	$\frac{15}{6}$	$\frac{8}{4}$

- a. Write down two matrices **P** and **Q** such that the elements of the matrix product  $\mathbf{A} = \mathbf{PQ}$  give the total cost of the bags delivered to each store. Find the value of **A**. (3)
- b. Write down two matrices **M** and **N** such that the elements of the matrix product  $\mathbf{B} = \mathbf{PQ}$  give the total amount that each store charged for the bags they sold. (3)
- c. Find  $\mathbf{B} - \mathbf{A}$  and explain what it means in the context of the problem. (2)

# Diving In

## AP Precalculus



### 5.5) Linear Transformation/Matrices (Calculator)

1. Given the vectors shown, the linear transformation using the given modification matrix  $M$  to create it will have an area in what relation to the original?

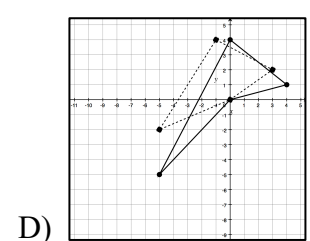
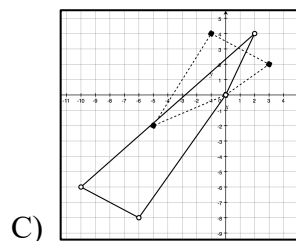
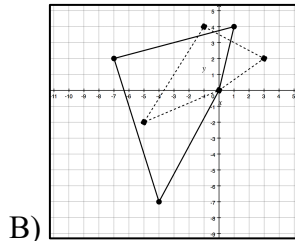
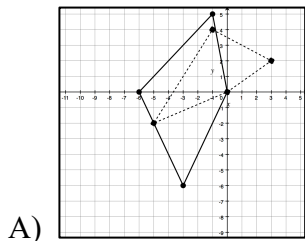
$$A = \begin{bmatrix} -2 \\ -2 \end{bmatrix}, B = \begin{bmatrix} 4 \\ 3 \end{bmatrix}, M = \begin{bmatrix} -1.5 & 3 \\ -2 & 2 \end{bmatrix}$$

- A) 3 times as large      B) 9 times as large      C) twice as large      D) 1/2 as large

$$A. \begin{bmatrix} -1.5 & 3 \\ -2 & 2 \end{bmatrix} = |-3 + 6| = 3$$

2. A matrix is  $\begin{bmatrix} 3 & -1 & -5 \\ 2 & 4 & -2 \end{bmatrix}$  is shown on the figures below as dotted lines. It is modified by the matrix

$$M = \begin{bmatrix} 2 & -2 \\ 2 & -1 \end{bmatrix}. \text{ Which of the following shows the transformation matrix in solid lines?}$$



$$C. \begin{bmatrix} 2 & -2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 3 & -1 & -5 \\ 2 & 4 & -2 \end{bmatrix} = \begin{bmatrix} 2 & -10 & -6 \\ 4 & -6 & -8 \end{bmatrix}$$

3. A matrix transformed by  $180^\circ$  would have its transformation matrix as which of the following?

A)  $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

B)  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

C)  $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

D)  $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$$A. M = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \Rightarrow M_{180^\circ} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

4. A unit square that was rotated  $60^\circ$  clockwise and dilated a factor of 1.5 has a transition matrix as

A)  $\begin{bmatrix} \sqrt{6}/4 & -3\sqrt{2}/4 \\ 3\sqrt{2}/4 & \sqrt{6}/4 \end{bmatrix}$

B)  $\begin{bmatrix} \sqrt{6}/4 & 3\sqrt{2}/4 \\ -3\sqrt{2}/4 & \sqrt{6}/4 \end{bmatrix}$

C)  $\begin{bmatrix} -3\sqrt{2}/4 & \sqrt{6}/4 \\ \sqrt{6}/4 & 3\sqrt{2}/4 \end{bmatrix}$

D)  $\begin{bmatrix} 3\sqrt{2}/4 & \sqrt{6}/4 \\ \sqrt{6}/4 & -3\sqrt{2}/4 \end{bmatrix}$

$$B. \sqrt{\frac{3}{2}} \cos 300^\circ = \sqrt{\frac{3}{2}} \left( \frac{1}{2} \right) = \frac{\sqrt{3}}{2\sqrt{2}} = \frac{\sqrt{6}}{4} \quad \sqrt{\frac{3}{2}} \sin 300^\circ = \sqrt{\frac{3}{2}} \left( \frac{-\sqrt{3}}{2} \right) = \frac{-3\sqrt{2}}{4} \quad M = \begin{bmatrix} \sqrt{6}/4 & 3\sqrt{2}/4 \\ -3\sqrt{2}/4 & \sqrt{6}/4 \end{bmatrix}$$

5. A cruise leaves Sunday afternoon and returns the following Sunday morning. On Sunday night, 60% of passengers eat dinner in the main dining room. 35% eat dinner in the buffet and 5% eat dinner in a specialty restaurant. Every night, for those who eat in the main dining room, 80% will continue to eat in the main dining room, 5% will switch to the buffet, and 15% will eat in a specialty restaurant. For those who eat in the buffet, 75% will continue to eat in the buffet, 20% will switch to the main dining room, and 5% will eat in a specialty restaurant. For those who eat in a specialty restaurant, 50% will eat the next night in the main dining room, 5% will eat in the buffet, and 45% will eat in another specialty restaurant.

a) Create the initial state matrix and the transition probability matrix. (2)

$$S_0 = \begin{bmatrix} 0.6 & 0.35 & 0.5 \end{bmatrix} \quad P = \begin{bmatrix} 0.80 & 0.05 & 0.15 \\ 0.20 & 0.75 & 0.05 \\ 0.50 & 0.05 & 0.45 \end{bmatrix}$$

1 pt for initial state matrix  
1 pt for transition matrix

b) What is the prediction for the number of passengers in each venue Monday night? (3)

$$S_0 P = \begin{bmatrix} 0.575 & 0.295 & 0.130 \end{bmatrix}$$

On the 2nd night, the probability is that 57.5% will eat in the main dining room, 29.5% will eat in the buffet, and 13% will eat in a specialty restaurant.

1 pt for main dining room  
1 pt for buffet  
1 pt for specialty restaurant

c) What is the prediction of passengers in each venue on the last night of the cruise? (3)

$$S_0 P^6 = \begin{bmatrix} 0.627 & 0.188 & 0.185 \end{bmatrix}$$

On the 6th night, the probability is that 62.7% will eat in the main dining room, 18.8% will eat in the buffet, and 18.5% will eat in a specialty restaurant.

1 pt for main dining room  
1 pt for buffet  
1 pt for specialty restaurant