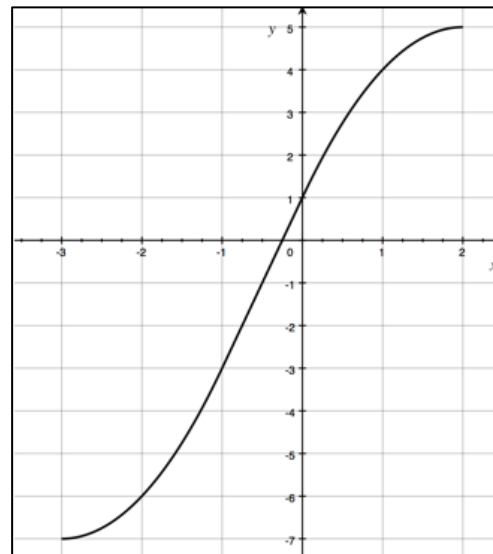


Advanced Placement Precalculus Sample Exam 1

Section I – Multiple Choice – Part A – Calculator Not Permitted – 80 Minutes

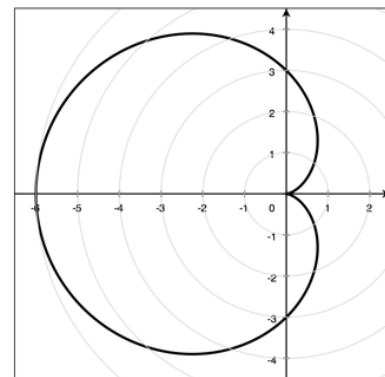
1. The interval that describes where  $f(x)$ , shown to the right, is concave up is which of the following?

- A)  $(-3, -1)$                       B)  $(-3, 0)$   
 C)  $(-1, 2)$                         D)  $(0, 2)$



2. The graph to the right could be which of the following?

- A)  $r = 2.5 - 3.5\cos\theta$             B)  $r = 2 - 4\cos\theta$   
 C)  $r = 3 + 3\sin\theta$                 D)  $r = 3 - 3\cos\theta$



3. Let the sequence  $a$  be defined as follows:  $a_1 = 100$   
 $a_n = a_{n-1} - 6$ . Find  $\sum_{i=1}^{25} a_i$ .

- A) 700                                  B) 550                                  C) 500                                  D) 400

4. Given that  $x = 3$  is a zero of  $f(x) = 2x^3 + x^2 - 25x + 12$ , which of the following best describes the nature of the other zeros of  $f$ ?

- A) 2 integers                          B) 2 rational                          C) 2 irrational                          D) 2 imaginary

5. Find the value of  $\left(-2 \tan \frac{3\pi}{4} - 2 \sin \frac{5\pi}{3} - \cos^2 3\pi\right)^2$

A)  $4 - 2\sqrt{3}$

B)  $7 - 4\sqrt{3}$

C)  $4 + 2\sqrt{3}$

D)  $7 + 4\sqrt{3}$

6. Ms. Moneypenny is a stockbroker. When she gets new clients, she first asks them their financial goals. Let  $r$  be how the client wants to increase their investment ( $r = 2$  for doubling,  $r = 3$  for tripling, etc.) and  $n$  be the number of years they desire for it to do so. What is the formula in terms of  $r$  and  $n$  that would generate an interest rate  $I$  stock increase that would allow the client to reach his goals if he invests  $P$  dollars?

A)  $\ln\left(\frac{n}{r}\right)$

B)  $\frac{\ln n}{r}$

C)  $\ln\left(\frac{r}{n}\right)$

D)  $\frac{\ln r}{n}$

7. Given  $\sin A = \frac{3}{5}$ ,  $A$  in quadrant II and  $\cos B = \frac{1}{3}$ ,  $B$  in quadrant IV, find  $\cos(B - 2A)$

A)  $\frac{7 + 48\sqrt{2}}{75}$

B)  $\frac{7 - 48\sqrt{2}}{75}$

C)  $\frac{-24 + 14\sqrt{2}}{75}$

D)  $\frac{-24 - 14\sqrt{2}}{75}$

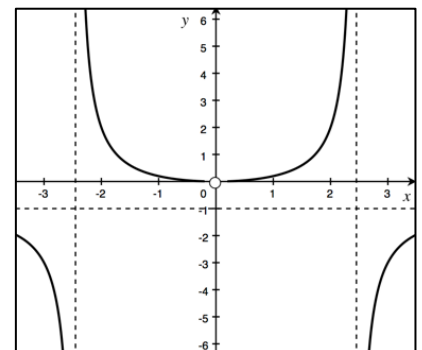
8. Which of the following equations could graph the function on the right?

A)  $y = \frac{-x^2}{x^2 - 6x}$

B)  $y = \frac{x}{x^2 - 6}$

C)  $y = \frac{x^3}{6x - x^3}$

D)  $y = \frac{2x^3}{x^3 - 6.25x}$



9. Let  $P(-6, 8)$  be a point on the terminal side of  $\theta$ . Place the following in descending order of value.

I.  $\sin\theta - \cos\theta$

II.  $\cot\theta - \tan\theta$

III.  $\csc\theta - \sec\theta$

A) I, III, II

B) II, I, III

C) III, I, II

D) I, II, III

10. Solve for  $x$ :  $40^{2x-3} = 2^{6x+1}$

I.  $x = \frac{3\log 40 + \log 2}{2\log 40 - 6\log 2}$

II.  $x = \frac{3 + 3\log 4 + \log 2}{2 + 2\log 4 - 6\log 2}$

III.  $x = \frac{3 + 7\log 2}{2 - 2\log 2}$

A) I only

B) I and II only

C) I, II and III

D) None of these

11. Let  $y = \frac{x^3 + 3x^2 - x - 3}{x^2 - 4}$ . Which of the following describes the graph's behavior?

I. Vertical asymptotes at  $x = 2, x = -2$

II. Oblique asymptote at  $y = x + 3$

III. Horizontal asymptote at  $y = 1$

IV. Zeros at  $x = \pm 1$  only

A) I only

B) I and II only

C) I, II, and IV only

D) III and IV only

12. Let  $f(x) = x^2 + 9$  and  $g(x) = x + 2$ . For what value of  $x$  does  $(g \circ f)(x) = (f \circ g)(-x)$ .

A)  $\frac{1}{2}$

B)  $-\frac{1}{2}$

C)  $-2$

D) no real values

13. If  $f(x) = -2x^2 - 5x + 3$ , which of the following could represent the formula for the average rate of change of  $f$  between  $x = a$  and  $x = a + 1$ ?

A)  $y = -4a - 7$

B)  $y = -4a + 3$

C)  $y = -7a - 11$

D)  $y = -7a - 7$

14. If  $f(x) = \left(\frac{1}{27}\right)^x$ , the graph of  $f\left(\frac{1-x}{3}\right)$  is the same as the graph of

A)  $y = 3^x$  shifted 1 unit down

B)  $y = 3^x$  shifted 1 unit up

C)  $y = 3^x$  shifted 1 unit to the left

D)  $y = 3^x$  shifted 1 unit to the right

15. Guido's Pizzeria offers pizzas in 3 sizes. As the size changes, the cost changes. Guido's offers a personal pizza with diameter 10 inches costing \$12 and a medium size with diameter 14 inches costing \$16. If the large pizza has diameter 16 inches, what should its cost be if the rate of change of its cost with respect to size remains consistent?

A) \$18

B) \$18.33

C) \$18.50

D) \$18.75

16. An exponential curve passes through the points  $P(0, 96)$  and  $Q(6, 12)$ . Find the value of  $y$  when  $x = 14$ .

A)  $\frac{3}{2}$

B)  $\frac{3}{4}$

C)  $\frac{3}{8}$

D)  $\frac{1}{4}$

17. If  $f(x) = \frac{3x-2}{2x+1}$ , find  $f^{-1}(-3)$

A)  $\frac{11}{5}$

B)  $\frac{5}{11}$

C) 1

D)  $\frac{-1}{9}$

18. Let  $f(x) = 2x^6 + 20x^4 + 50x^2$ . For  $f$ , let  $d$  equal the degree of the equation,  $q$  be the number of unique complex zeros, and  $r$  be the sum of the multiplicities of each linear factor. What is the relationship between  $p$ ,  $q$ , and  $r$ ?

A)  $d = q + r$

B)  $d - q - r = 1$

C)  $d = 2q + r$

D)  $d = r^q$

19. Solve for  $x$ :  $\frac{-2x^2 + 20}{x^2 - 4x + 4} \geq 0$

A)  $[-\sqrt{10}, \sqrt{10}]$

B)  $[-\sqrt{10}, 2), (2, \sqrt{10}]$

C)  $(-\infty, -\sqrt{10}], [\sqrt{10}, \infty)$

D)  $(-\infty, -\sqrt{10}], (2, \infty)$

20. The fall semester of a college is 10 weeks long. The college's dining hall managers wish to develop a formula that measures how many pounds of cheese it uses for its pizza as a function of the week number using regression. They first need to determine the type of regression to use. Based on the table below giving both the difference and ratio of successive cheese data values, which regression method makes the most sense?

<b>Week Number</b>	1	2	3	4	5	6	7	8	9	10
<b>Pounds of Cheese</b>	160	184	209	237	267	300	335	372	411	453
Difference of Successive Data	–	23	25	28	30	33	35	37	39	42
Ratio of Successive Data	–	1.146	1.140	1.134	1.128	1.122	1.116	1.111	1.106	1.101

- A) Linear                      B) Quadratic                      C) Exponential                      D) Logarithmic

21. Place in descending order in terms of value.

I.  $\log_9 \sqrt{3}$

II.  $e^{2\ln(1/3)}$

III.  $2\log_4 \frac{1}{32}$

IV.  $\left(\log \frac{1}{\sqrt[3]{100}}\right)^2$

- A) III, IV, I, II

- B) III, II, IV, I

- C) IV, I, II, III

- D) IV, II, III, I

22. Find the product of the roots of  $f(x) = \log_2(x^2 - 6x - 12) - \log_2(-1 - x) - 2$ .

- A) -2

- B) 2

- C) 4

- D) -8

23. How many of the following equations graph the figure on the right?

i.  $y = 1 - 2\cos 2x$

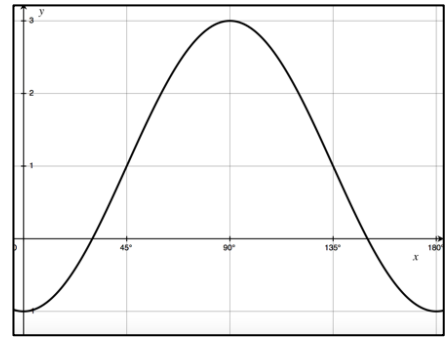
ii.  $y = 1 + 2\cos(2x - \pi)$

iii.  $y = 1 + 2\sin\left(2x - \frac{\pi}{2}\right)$

iv.  $y = 1 + 2\sin\left(2x + \frac{\pi}{2}\right)$

v.  $y = 1 - 2\cos\frac{x}{2}$

vi.  $y = 1 + 2\sin\left(\frac{1}{2}x - \pi\right)$



A) 4

B) 3

C) 2

D) 0

24. Which of the following equations has a solution:  $x = \frac{\pi}{6} + \frac{2\pi n}{3}$ , with  $n$  an integer?

I.  $\cos 2x = \sin x$

II.  $\cos^2 x = \frac{3}{4}$

A) I only

B) II only

C) I and II

D) Neither

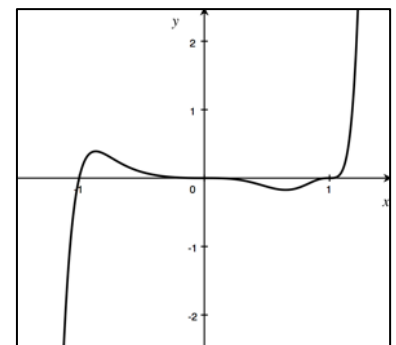
25. Match the graph to the right to the only possible function.

A)  $y = (x+1)(x-x^4)^3$

B)  $y = -(x+1)(x-x^4)^3$

C)  $y = (x-1)(x+x^3)^3$

D)  $y = -(x-1)(x+x^3)^3$



26. Use polar form to find the value of  $(-\sqrt{3} + i)^5$

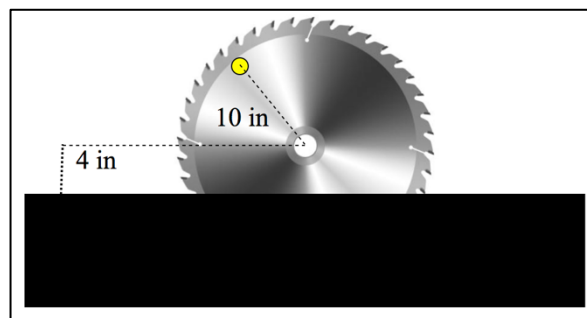
A)  $16\sqrt{3} + 16i$

B)  $-16\sqrt{3} - 16i$

C)  $16\sqrt{3} - 16i$

D)  $-16\sqrt{3} + 16i$

27. A sawmill has a saw for cutting logs. The saw is inset in a platform for the logs to be split as shown to the right. A yellow dot is painted on the edge of the blade, 10 inches from the center of the blade which is 4 inches off the platform. If the blade turns at 1800 revolutions per minute, which of the following could be the height in inches of the yellow dot relative to the platform at time  $t$  seconds?



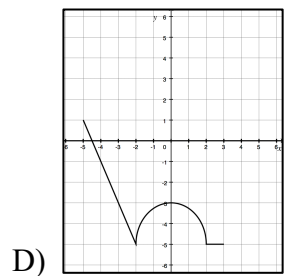
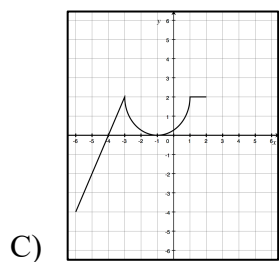
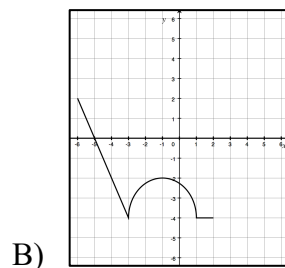
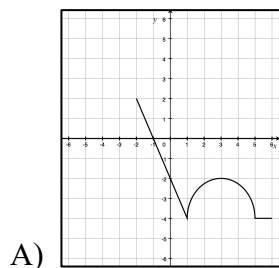
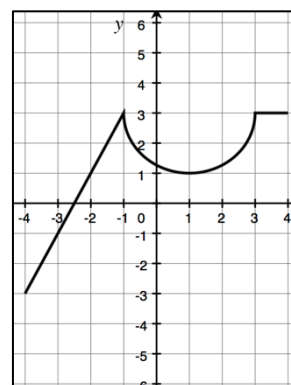
A)  $y = 14\cos 1800\pi t$

B)  $y = 4 + 10\cos 120\pi t$

C)  $y = 4 + 10\cos \frac{\pi t}{120}$

D)  $y = 4 + 10\cos 60\pi t$

28. The graph of  $f(x)$  as shown to the right is defined for  $-4 \leq x \leq 4$  and consists of 2 line segments and a semi-circle. Which of the following represents the graph of  $y = -1 - f(x+2)$ ?



Name \_\_\_\_\_ Period \_\_\_\_\_

**Advanced Placement Precalculus Sample Exam 1**

**Section I – Multiple Choice – Part B – Calculator Required – 40 Minutes**

29. Find the sum:  $1600 - 1600\left(\frac{3}{4}\right) + 1600\left(\frac{3}{4}\right)^2 - 1600\left(\frac{3}{4}\right)^3 + \dots + 1600\left(\frac{3}{4}\right)^{20}$

A) 2,140.099

B) 2,138.407

C) 916.460

D) 917.185

30. The cost of a gym membership at the start of the year 2020 was \$35/month. The costs are actually decreasing by 3.3% every 2 years with the price announced at the beginning of the year. In what year is the cost projected to be \$30/month or less?

A) 2024

B) 2025

C) 2029

D) 2030

31. The number of people in a cruise ship buffet behaves in a roughly periodic fashion with a low of 120 people at 7:00 AM and a high of 1,040 people at 8:30 AM. Let  $t$  be the hour of the day with  $t = 0$  corresponding to 12 AM. What is the projected difference of people at the buffet between 7:15 AM and 8:00 AM?

A) 460

B) 602

C) 628

D) 667

32. Let  $f(x) = x^2 - 3x + 1$ . For what value of  $n$  could the instantaneous rate of change of  $f$  at  $x = n$  equal the average rate of change of  $f$  between  $x = -1$  and  $x = 5$ ?

- A) 0                                      B) 1                                      C) 2                                      D) 2.5

33. A patient takes a strong pain medication every 4 hours for cellulitis pain. He is asked to monitor his pain on a 0-10 basis (10 being the worst) at different intervals of time. Using the data below, generate a periodic function that describes that models his pain as a function of time and use it to predict his pain at 4 PM if he continues with this dosage.

Time (AM)	8:00	8:15	8:30	8:45	9:00	9:20	9:40	10:00	10:15	10:40	11:00	11:30	12:00
Pain	5	7	8	6	4	2.5	2	3.5	7	8	7	4	3.5

- A) 2.7 and decreasing              B) 3.1 and increasing              C) 4.2 and increasing              D) 4.2 and decreasing

34. Proud Home Warranty covers all appliance repairs and replacements with a yearly subscription. Here are the terms for purchasing the subscription.

Pay \$39/month and \$45 per service call for up to 5 service calls

Pay \$15 per service call after 5 service calls

Which represents a function describing the yearly cost based on the number of service calls  $x$ ?

A)  $f(x) = \begin{cases} 39 + 45x, & x \leq 5 \\ 263 + 15x, & x > 5 \end{cases}$

B)  $f(x) = \begin{cases} 39 + 45x, & x \leq 5 \\ 263 + 15(x - 5), & x > 5 \end{cases}$

C)  $f(x) = \begin{cases} 468 + 45x, & x \leq 5 \\ 468 + 15(x - 5), & x > 5 \end{cases}$

D)  $f(x) = \begin{cases} 468 + 45x, & x \leq 5 \\ 618 + 15x, & x > 5 \end{cases}$

35. The gangplank to the entry door of a cruise ship is at an angle of  $18^\circ$ . Passengers complain it is too steep so the crew lengthens it and moves it back 50 feet and the angle is now  $11^\circ$ . How high is the ship's entry door above the dock?
- A) 21.5                      B) 24.2 ft                      C) 28.9 ft                      D) 36.8 ft
36. Steve sets out on a weeklong vacation to a sunny island. On the first day, he notices that he has a rash that he estimates that covers 1.5% of his right arm. Two days later, the rash has spread and now the estimation is it covering 3.2% of his arm. He uses a linear and exponential model to predict the difference in spread of the rash by the time the vacation ends. If he does nothing to treat it, what is the expected difference in these models?
- A) 8.0%                      B) 13.8%                      C) 21.3%                      D) 22.8%
37. In an advance sale, a concert promoter will sell out his 1,200-seat auditorium if he sells tickets for \$50. For every \$5 he increases his price, he will sell 50 fewer tickets. On the day of the concert, he sells all unsold seats at \$20 per ticket. What should be the advance price for a ticket if the promoter wants to maximize his revenue?
- A) \$70                      B) \$85                      C) \$95                      D) \$102.50

38. The polar form of  $i$  can be expressed as  $\cos 90^\circ + i\sin 90^\circ$ . What is the magnitude of  $\sqrt{i^3\sqrt{i}}$ ?

A) 0.258

B) 0.966

C) 1

D) 1.224

39. Describe the graph of the inverse to the function  $y = 3 - 4\sin(x - 1)$ .

A) Concave up

B) Concave down

C) Concave down  
then concave up

D) Concave up  
then concave down

40. Ant baits contain insecticides mixed with materials that attract ants looking for food. Ants are attracted to the bait and recruit other ants to it. Ants carry small portions of the bait back to the nest where it is transferred mouth to mouth to other ants to kill the entire colony. Bait products must be slow-acting so that the foraging ants have time to make their way back to the nest and feed other colony members.

The effectiveness  $E$  of an ant bait is measured on a 0 to 1 scale. The greater the value of  $E$ , the more effective the bait is. The effectiveness of a particular bait product  $d$  days after applying it is given by

$$E(d) = \frac{12d^2 + 4d}{d^4 + 75}, d > 0. \text{ How many of the following statements are true?}$$

I. The bait is most effective between day 2 and day 3.

II. The bait effectiveness is increasing the fastest between day 1 and day 2

III. The bait's effectiveness is decreasing the fastest between day 5 and day 6

IV. The average rate of effectiveness of the bait from the day of application to any day after that will always be positive.

A) 4

B) 3

C) 2

D) 1

Name \_\_\_\_\_ Period \_\_\_\_\_

**Advanced Placement Precalculus Sample Exam 1**

**Section II – Free Response – Part A – Calculator Permitted – 30 Minutes**

1. Let  $f(x) = x^5 + x^4 - x^3 - 1$  and  $g(x) = \frac{f(x)}{x}$

a. Complete the table and use the average rate of change (AROC) analysis at each value of  $x$  in the table to explain why  $g$  could have an inflection point at  $x = 0$ .

$x$	-0.2	-0.1	0.0	0.1	0.2
$g(x)$					
AROC					

b. Determine any vertical asymptotes of  $g$ .

c. Determine the full factorization of  $g$ .

d. Find the end behavior of  $g$ . Explain why this supports this function not having a horizontal asymptote.

2. On the day a cruise ship goes through the Panama Canal, ship staff placed information packets on its decks as passengers get up very early to see the spectacle. At 6 AM, there were 1,000 packets while at 8 AM, there were 450 packets.
- Using a linear model of time versus packets, predict how many packets  $P$  were initially on deck at 5 AM. Show how you get your answer.
  - Using time zero as 6 AM, use your calculator to generate an exponential function that describes the number of packets on deck at 5 AM. Show your equation.
  - Show that an equivalent equation for part b above can be generated without using the calculator.
  - At 9 AM, there are 260 packets remaining. Perform quadratic regression with the 3 data points and use it to predict the difference in packets remaining using the exponential model at 12 noon.
  - Explain which model makes the most sense for predicting packets on deck throughout the day.



4. Let  $z = 64(\cos 75^\circ + i \sin 75^\circ)$

a. Find  $|z|$

b. Write  $z$  as a complex number  $a + bi$  with  $a$  and  $b$  real numbers

c. Use the answer in b) to verify the answer in a).

d. Without calculating them, determine the quadrants in the complex plane of the cube roots of  $z$ . Show how you get your answers.