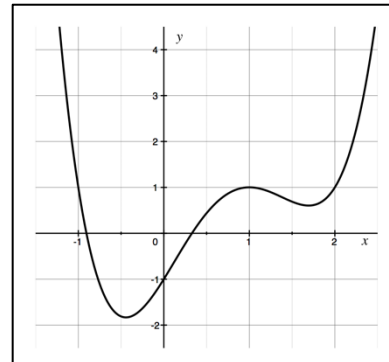


Advanced Placement Precalculus Sample Exam 2

Section I – Multiple Choice – Part A – Calculator Not Permitted – 80 Minutes

1. The graph of $g(x)$ is to the right. Let a be the number of real zeros, b : the number of relative minimums, c : the number of relative maximums, d : the number of absolute minimums, e the number of absolute maximums, and g : the number of inflection points. Find the value of $a + b + c + d + e + f$



- A) 6
B) 7
C) 8
D) 9
2. Suppose θ is a quadrant III angle. Which of the following must have the largest value?

- A) $\sin\theta\cos\theta$ B) $\csc\theta\sec\theta$ C) $\tan\theta\cot\theta$ D) $\sin\theta\sec\theta$

3. Let $f(x) = \sqrt{\frac{1}{2}x - 2}$ and $g(x) = \frac{x}{x-6}$. Find the domain of $f \circ g$?

- A) $[4, 8], x \neq 6$ B) $x \geq 4, x \neq 6$ C) $[4, 6)$ D) $(6, 8]$

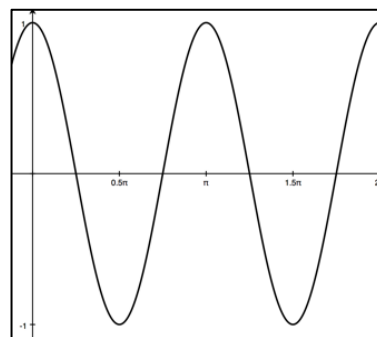
4. Let $f(x) = \frac{x}{x+2} + \frac{1}{x}$. Determine how many points to assign the function if 1 point is given for each real zero, 1 point for each vertical asymptote not on the y -axis, one point for any horizontal asymptote not on the x -axis, and 1 point for each y -intercept.

- A) 2 B) 3 C) 4 D) 6

5. An equation for the graph on the right could be

- I. $y = \cos 2x$ II. $y = -\sin\left(x - \frac{\pi}{4}\right)$ III. $y = 1 - 2\sin^2 x$

- A) I only B) I and II only
 C) I and III only D) II and III only



10. If a is a positive real number, what is $\log 4 \cdot \log_8 a$

- A) $\frac{2}{3} \log a$ B) $\frac{1}{2} \log a$ C) $\frac{1}{2} \log_8 8a$ D) $\frac{1}{2} \log 8a$

7. Given $\sin A = \frac{2}{3}$, $90^\circ < A < 180^\circ$, which of the following is true?

- I. $\tan 2A = -4\sqrt{5}$ II. $\tan 2A = 4\sqrt{5}$ III. $2A$ is in quadrant III IV: $2A$ is in quadrant IV

- A) I and III B) I and IV C) II and III D) II and IV

8. If the expression $(x-2)^7 + (2x^2-3)^5$ is expanded, what is the coefficient of the x^6 term?

A) $58x^6$

B) $65x^6$

C) $706x^6$

D) $-724x^6$

9. Find $\sec\left(\sin^{-1}\frac{-1}{5}\right)$

A) $\frac{-5\sqrt{6}}{12}$

B) $\frac{5\sqrt{6}}{12}$

C) $-2\sqrt{6}$

D) $2\sqrt{6}$

10. A population of a particular species of fish off an island follows a periodic pattern due to water temperature. The number of fish is given by the function $f(t) = 60000 + 3600\cos t$. Starting with time $t = 0$ months at the beginning of January, how times during the first 4 months of the year is the fish population 61,800 fish?

A) 0

B) 1

C) 2

D) 3

11. Let $f(x) = 5x^2 + ax$, a a real number. An average rate of change formula is to be constructed that can be used to find the average rate of change of f between any two odd integers. What is that formula?

A) $y = (a+15)x$

B) $y = (a+10)x$

C) $y = 15x + a + 5$

D) $y = 10x + a + 10$

12. $(\csc\theta + \cot\theta)(1 - \cos\theta) = \sin\theta$ is true for which of the following angles?

I. $\theta = \frac{\pi}{4}$

II. $\theta = \frac{5\pi}{6}$

III. $\theta = \frac{3\pi}{2}$

A) I and II only

B) I and III only

C) II and III only

D) I, II and III

13. Use partial fraction techniques to rewrite $\frac{x+a+4}{x^3-8x^2+15x}$ with a a real number.

A) $\frac{4a}{15x} + \frac{9a}{10(x-5)} - \frac{7a}{6(x-3)}$

B) $\frac{a+4}{x} + \frac{a+9}{x-5} + \frac{a+7}{x-3}$

C) $\frac{a+4}{x} + \frac{a+9}{x-5} - \frac{a+7}{x-3}$

D) $\frac{a+4}{15x} + \frac{a+9}{10(x-5)} - \frac{a+7}{6(x-3)}$

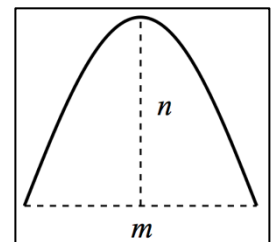
14. The figure to the right is part of a sinusoid. For which function is $m > n$?

A) $y = 4 + 2\cos 2x$

B) $y = 3 - 8\sin \frac{1}{2}x$

C) $y = -4\sin\left(\frac{2}{3}x - \pi\right)$

D) $y = 1 - \frac{3}{4}\cos\left(6x - \frac{\pi}{2}\right)$



15. Fred's Flights charts small airplanes. The cost for the plane is \$500 and the charge for pilot time is as follows: \$150/hour for the first 2 hours, \$125/hour for the 3rd hour or part, and \$100/hour for flights over 3 hours. There is also an option for paying \$1,200 for unlimited time. How many hours would a flight have to be to make paying the unlimited time fee preferable for the customer? Choose the answer that describes all of the times.

- A) over 5.5 hours B) over 5.75 hours C) over 6 hours D) over 6.25 hours

16. Find the distance between the roots of $f(x) = \log_3(x^2 + x - 3) - 2 - \log_3(x - 2)$. If there is only one root or no roots, use answer (A).

- A) 0 B) 1 C) 2 D) 3

17. Solve for x : $\frac{8}{(x-6)^2} \geq \frac{4}{6-x}$

- A) $[4, 6), (6, \infty)$ B) $[4, 6)$ C) $[4, \infty)$ D) $(-\infty, 6]$

18. A 4th-degree polynomial has roots of $1 - 2i$ and $1 - \sqrt{2}$. Which of the following is its factorization?

- A) $(x^2 + 5)(x^2 - 2x + 3)$ B) $(x^2 - 2x + 5)(x^2 - 2x + 3)$
C) $(x^2 + 5)(x^2 - 2x - 1)$ D) $(x^2 - 2x + 5)(x^2 - 2x - 1)$

19. A quadratic function generates the following values. Find the value of $f(2.5)$.

x	-2.5	-2.0	-1.5	-1.0	-0.5	0.0
$f(x)$	16	9	3.5	-0.5	-3	-4

A) 1.5

B) 13.5

C) 16

D) 20

20. Let $M = 2(\cos 15^\circ + i \sin 15^\circ)$, Find the value of $M^4 - M^2$.

A) $6\sqrt{3} + 6i$

B) $2\sqrt{3} + 2i$

C) $(4 - 2\sqrt{3}) + i(4\sqrt{3} - 2)$

D) $(8 - 2\sqrt{3}) + i(8\sqrt{3} - 2)$

21. Which of the following has a difference result than the others.

A) $\log_8 \frac{1}{16}$

B) $\log \frac{1}{\sqrt[3]{10000}}$

C) $\log_{8/27} \frac{9}{4}$

D) $\frac{-4}{9e^{\ln(1/3)}}$

22. A function passes through the points (3, 18) and (5, 162). Find the difference between a linear and exponential model for the function at $x = 4$.

A) 18

B) 36

C) 46

D) 54

23. Which of the following is equivalent to $\cos 3\theta - i \sin 3\theta$?

A) $\cos^3 \theta - i \cos^2 \theta \sin \theta - \cos \theta \sin^2 \theta + i \sin^3 \theta$

B) $\cos^3 \theta - 3i \cos^2 \theta \sin \theta - 3 \cos \theta \sin^2 \theta + i \sin^3 \theta$

C) $\cos^3 \theta + i \cos^2 \theta \sin \theta + \cos \theta \sin^2 \theta + i \sin^3 \theta$

D) $\cos^3 \theta + 3i \cos^2 \theta \sin \theta + 3 \cos \theta \sin^2 \theta + i \sin^3 \theta$

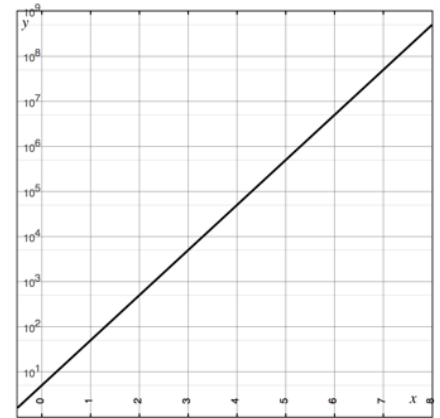
24. The graph of a function is shown on a semi-log plot as shown in the figure to the right. What is the function?

A) $y = 10^x - 5$

B) $y = 10^x - 3$

C) $y = 5 \cdot 10^x$

D) $y = 7 \cdot 10^x$



25. A cubic function is in the form $f(x) = ax^3 + bx^2 + cx + d, a > 0$ has 3 real distinct zeros. Which of the following **must** be true of the function?

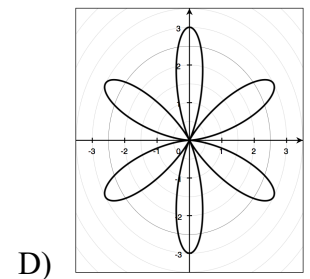
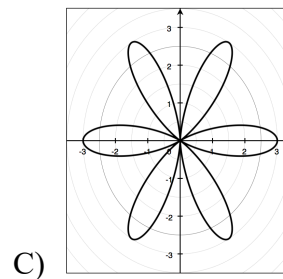
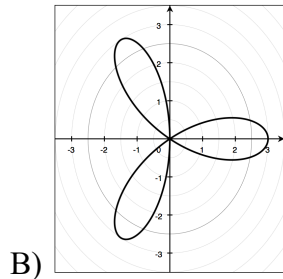
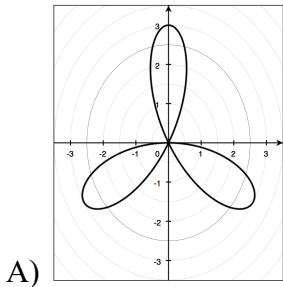
A) f is always increasing

B) f has an absolute maximum

C) f has a relative maximum and a relative minimum

D) f has no inflection points

26. Which of the following is the graph of $r = 3\cos\left(3\theta + \frac{\pi}{2}\right)$?



27. An athlete is working out on the steps of a stadium. The lowest row of seats, row 20 is reached in 36 seconds. The highest row, row 84, is reached in 1 minute, 18 seconds. If she works out based on simple harmonic motion, what is a possible equation that describes the row R she is passing as a function of t in seconds?

A) $R = 52 - 32\cos\frac{\pi}{42}(t - 36)$

B) $R = 52 + 32\sin\frac{\pi}{42}(t - 78)$

C) $R = 52 - 32\sin\frac{\pi}{42}(t - 57)$

D) $R = 52 - 32\cos\frac{\pi}{42}(t - 78)$

28. Which of the following functions could create the graph to the right?

i. $y = -(x-1)^3(x+2)^2$

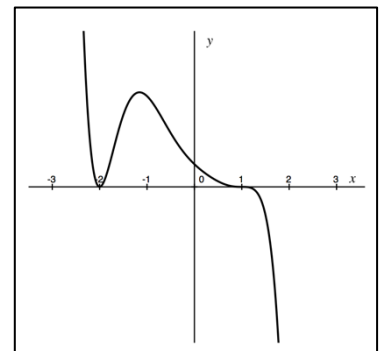
ii. $y = -(x-1)^3(x^2+1)(x+2)^2$

iii. $y = (x-1)^2(x+2)^2$

iv. $y = -(x-1)^2(x^2+1)(x+2)^2$

- A) i only
C) iii only

- B) i and ii only
D) iii and iv only



Name _____ Period _____

Advanced Placement Precalculus Sample Exam 2

Section I – Multiple Choice – Part B – Calculator Required – 40 Minutes

29. Which of the following functions is one-to-one?

I. $y = 2x^5 + x$ II. $y = x^3 + x^2, x \geq -1$ III. $y = \sqrt[3]{2x^3 + x^2 + 0.6}$

- A) I only B) II only C) III only D) I and III only

30. Jennifer is practicing the Minute Waltz. She takes 3 minutes to play it and immediately starts again. Each time she plays, she reduces her time by 9 seconds. If she practices for a half hour, how many times will she play the waltz?

- A) 12 B) 16 C) 20 D) 25

31. A politician is having a huge rally and gives out free hats. At $t = 1$ hour there are 3,000 hats available for distribution. At $t = 4$, there are only 600 hats available. Which could be the function: time t in hours vs. hats h available if the graph is concave down?

- A) $h(t) = 3800 - 800t$ B) $h(t) = 400t^2 - 2800t + 5400$
C) $h(t) = 3000e^{-0.536t}$ D) $h(t) = 3000\left(\sqrt[3]{0.2}\right)^{t-1}$

32. You are given the series $100\left[1 - \frac{2}{3} + \left(\frac{2}{3}\right)^2 - \left(\frac{2}{3}\right)^3 + \dots\right]$. If m is the sum of infinite series and n is the sum of the first 7 terms of the series, find $m - n$.

A) -3.512

B) 2.341

C) 5.368

D) 17.588

33. An earthquake in a remote region causes a huge need for blood. For the first 7 days, the amount of blood, measured in pints, that the Red Cross has in its stores is given by $b(t) = 1250 - 480\ln(t+1)$, t measured in days. The need for blood eases off slightly and the amount of blood it has in its stores is then given by $b(t) = 979 - 350\ln(t+1)$. What is the average rate of blood consumption given in pints/day from the beginning of the crisis until the Red Cross runs out of blood? (nearest pint)

A) 143

B) 81

C) 61

D) 29

34. On a cruise ship, the ship must be at least 3 miles from a port in order for the ship's casino to open. Many people will congregate in the casino waiting for the moment when the ship is the appropriate distance and slot machines are allowed to operate. The number of people in a particular ship's casino one night is given by the function $f(t) = \frac{10(3t+8)}{1+0.1t}$, t measured in minutes. How many minutes does it take for the number of people in the casino to increase from 85% to 95% of its maximum population?

A) between 30 minutes and an hour

B) between an hour and 90 minutes

C) between 90 minutes and two hours

D) over two hours

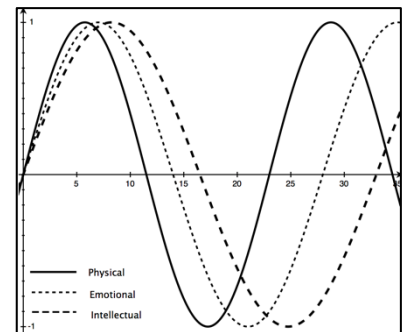
35. A traffic light turns from red to green and a motorcycle accelerates from a stop covering distance according to $d(t) = 3t^2 + 20t$, t measured in seconds and d measured in feet. What is the approximate difference between the average speed of the vehicle over the first 10 seconds and the instantaneous speed of the vehicle at $t = 10$ seconds (nearest integer)?

- A) 0 ft/sec B) 30 ft/sec C) 42 ft/sec D) 75 ft/sec

36. You are given a polar point $A = 3 + 4i$. You multiply A by $-1 + 2i$. How has A been transformed?

- A) 116.57° counterclockwise B) 63.43° clockwise
 C) 26.57° clockwise D) 153.43° counterclockwise

37. The theory of biorhythms (developed in the last 19th century) states that we go through periodic cycles of highs and lows in three areas of our lives. The physical cycle lasts 23 days, the emotional cycle lasts 28 days, and the intellectual cycle lasts 33 days, all starting at birth. These can be modeled by sinusoidal curves as shown in the figure to the right.



The composite score is the sum of the 3 cycles and the maximum indicates when we would have a good day and the minimum when we would have a terrible day. In the first half-year of a baby's life, how many days pass between its best day and its worst day?

- A) 13 B) 91 C) 151 D) 183

38. The last night of a cruise is lobster night in the restaurants. Many people order them despite their small size. The half-life of their supply of lobsters in the ship's kitchens is 37 minutes and passengers are told that they should make early reservations to be sure lobsters will still be available. If the ship has 2,500 lobsters when the restaurant opens at 5:30 PM, at what time of night will there be fewer than 50 lobsters left?
- A) 6:56 PM B) 8:59 PM C) 9:14 PM D) they won't run out

39. A student with no previous typing experience takes a three-week typing course and after each day takes a test. The test requires the student to type for 3 minutes with 98% accuracy. The speed is recorded in words per minute (wpm). Here is the data. Day 0 refers to a pre-test taken before the course started. Both linear regression and quadratic regression are performed on that data. Which of the following statements are true?

Day	0	1	3	6	7	10	12	14	16	18	19	21
Speed (wpm)	7	14	23	36	42	49	56	60	65	67	68	72

- I. For linear regression, every day averages a 3.02 word per minute increase in speed.
 II. For quadratic regression the residual for day 16 is positive.
 III. Assume that the student gets a secretarial job and continues typing. The quadratic model is a better predictor of the typing speed than the linear model one week later.
- A) I and II only B) I and III only C) II and III only D) I, II and III

40. A restaurant's profit on a Saturday night is dependent on its reservations. Too few and it doesn't make much profit. Too many and extra wait staff must be hired, reducing profit. The profit function for the restaurant is given by $P = -0.0023r^3 + 0.396r^2 + 0.47r - 5$. If the number of customers that gives a maximum profit is booked that night and the manager decides to take 5 more reservations, what is the average rate of change of profit for these 5 extra reservations?
- A) $\frac{-\$1.98}{\text{reservation}}$ B) $\frac{-\$9.92}{\text{reservation}}$ C) $\frac{-\$2.00}{\text{reservation}}$ D) $\frac{-\$10.01}{\text{reservation}}$

Name _____ Period _____

Advanced Placement Precalculus Sample Exam 2

Section II – Free Response – Part A – Calculator Permitted – 30 Minutes

1. A quadratic function f has roots of $x = -2$ and $x = \frac{1}{2}$.
- a. If the average rate of change of f between $x = -1$ and $x = 4$ is -27 , find $f(x)$.
- b. The vertex of the parabola is at $(-0.75, f(-0.75))$. Show that the instantaneous rate of change of f at $x = -0.75$ is zero.
- c. Using relative extrema theory and limits, what point on the function is $\left(-\frac{3}{4}, f\left(-\frac{3}{4}\right)\right)$? Justify your answer.
- d. Let $g(x) = [f(x)]^2$. Determine intervals where g is increasing and g is decreasing.
- e. The inflection points of g are at $x = -1.472$ and $x = -0.283$. Students mistakenly believe that the inflection points of a polynomial always lie halfway between a relative maximum and a relative minimum. Using this mistaken theory, how far off will a student be in finding the inflection pts.

2. The day before Christmas, a large corporation is distributing free gifts for its workers at a holiday get-together starting at 9 AM. The gifts are on tables and one gift is given for each worker. At $t = 1$ hour, there are 330 gifts on the tables and at $t = 2$ hours there are 231 gifts on the tables.
- Assuming that the gifts are taken according to an exponential function g , show necessary work to determine function g .
 - Explain why $f(t) = 471 - g(t)$ represents the number of workers who picked up gifts as a function of t .
 - Determine the time of day when 400 workers have picked up their gift. Show how you got your answer.
 - Using $f(1)$ and $f(2)$, create a logarithmic model $h(t)$ that approximates f . Use h to find the approximate difference between the exponential and logarithmic model at 5 PM.

Advanced Placement Precalculus Sample Exam 2

Section II – Free Response – Part B – Calculator Not Permitted – 30 Minutes

3. Phoebe is riding on a Ferris wheel that travels at a constant rate. At $t = 5$ seconds, she is at the low point, 1 foot above the ground and at $t = 20$ seconds, she is at the high point 61 feet above the ground.

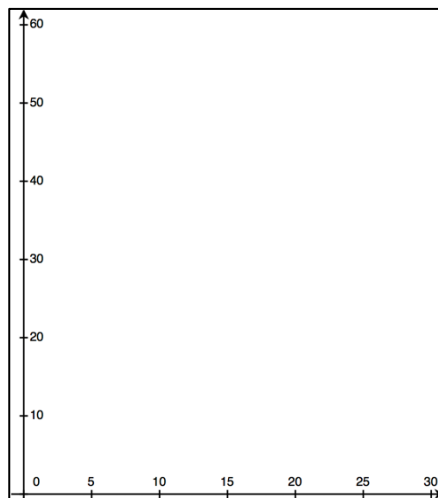
a. A polynomial function that approximates Phoebe’s height at $t = 20$ seconds is given by

$P(t) = 61 - \frac{2}{3}(t - 20)^2$. Use this function to approximate her average rate of change of height between $t = 20$ seconds and $t = 25$ seconds.

b. Write a periodic function that predicts her height H as a function of time t .

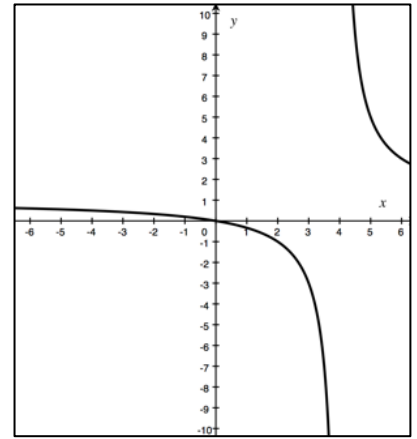
c. Using H , find the average rate of change in her height between $t = 20$ seconds and $t = 25$ seconds.

d. Sketch the two curves on the given grid and use the sketch to explain why $P(t)$ will give a good approximation of $H(t)$ only in a narrow range of t .



4. Let $f(x) = \frac{x}{x-4}$ and its graph appears to the right.

a. Use the graph to determine and explain whether f is one-to-one. Be sure to use asymptotes in your argument.



b. Find $f^{-1}(x)$.

c. If $g(x) = \sqrt{2x+2}$, find the domain of $g \circ f$. Explain your conclusion.

d. Find $(g \circ f^{-1})(-3)$