## *Why A.P. Precalculus?*

This is a quote from the Introduction of the new College Board AP Precalculus Course Framework.

> *During this course, students acquire and apply mathematical tools in real-world modeling situations in preparation for using these tools in college-level calculus. Modeling, a central instructional theme for the course, helps students come to a deeper understanding of each function type. By examining scenarios, conditions, and data sets, as well as determining and validating an appropriate function model, students develop a greater comprehension of the nature and behavior of the function itself. The formal study of a function type through multiple representations (e.g., graphical, numerical, verbal, analytical), coupled with the application of the function type to a variety of contexts, provides students with a rich study of precalculus.*
>
> *Throughout this course, students develop and hone symbolic manipulation skills needed for future mathematics courses. They also solve equations and manipulate expressions for the many function types throughout the course. Students also learn that functions and their compositions, inverses, and transformations are understood through graphical, numerical, verbal, and analytical representations, which reveal different attributes of the functions and are useful for solving problems in mathematical and applied contexts. In turn, the skills learned in this course are widely applicable in a variety of future courses that involve quantitative reasoning.*

The College Board makes it clear that this course is not meant for the student who knows he/she will be going into some STEM field and is following the AP Calculus route in high school. Rather, it is meant for those students who may not be entering a STEM field but will need a year of calculus in college. College instructors find that beginning calculus students are deficient in many skills and this course is meant to prepare them for that college course which can be difficult.

But moreover, the College Board is advocating a course that walks the line between the analytic skills required in calculus compared to one that is more relevant and engaging using data science. I believe that is a sensible approach and I am on board for making topics apply to real-life situations.

In developing an AP Precalculus curriculum, my first thought was to update the existing Precalculus manual I have on the MasterMathMentor.com website. The College Board website specifically states that teachers can use their existing textbooks to teach the course. However, in examining closely the learning objectives, I soon realized that the course that the College Board developed as their model was quite different from what I had been using all these years. And while a great number of topics were the same, they were tweaked in a way that I felt a major revision was necessary. And so the AP Precalculus manual was rewritten to conform to the new College Board approach and to make the transition to the new curriculum a smooth one for teachers and students.

Some examples of these revisions:

- It was apparent to me that this new course is starting to teach calculus even though it never formally states that it is doing so. There is a great amount of focus on average rate of change within their first chapter which even touches upon instantaneous rate of change. No mention is made of a derivative but the concept is hinted at, even to the point of introducing the concept of limits and its notation. Calculus develops a formula for the slope of the tangent line to a curve at any point, (i.e. the derivative). This course develops a formula for the average rate of change at an $x$-value. In the

discussion of functions, the concept of increasing and decreasing, relative maxima, concavity and inflection points, rate of change of rate of change (i.e., the second derivative) are stressed. While no attempt is made to find relative maxima or inflection points or areas of concavity mathematically, students are expected to recognize and approximate these concepts by examining a graph.

- A greater emphasis of technology is apparent. I had always subscribed to the Demana and Waits concept of "do mathematically, confirm graphically" approach and to rely on the calculator when it was difficult or impossible to solve a problem analytically. For instance, finding where an exponential curve intersects a linear function is a task that is better done graphically. However, the College Board includes all sorts of regression in this new AP Precalculus course. Linear regression, finding the line of best fit for a data set which is typically taught in AP Statistics, is a part of this course as well as exponential regression. Quadratic, logarithmic and sinusoidal regression, not taught in AP statistics, are also covered.

  It is a bit distressing to me in that we are using the calculator as a "black box." Students push the buttons and the regression equation appears without any understanding of how it was found and how accurate it might be. AP Statistics, while still using the calculator as a black box, focuses on the degree of reliability of these regression equations. AP Stat teaches the mechanics of hypothesis testing and only after students master it is the use of the calculator encouraged to crunch the numbers. That is a far different approach than AP precalculus takes.

  And yet, I can see the logic in teaching regression. Students press a sine button on their calculator without any real concept of what the calculator is doing to compute the sine of an angle. The sine function invokes Taylor polynomials, taught in BC Calculus and there is no way it would be appropriate to teach it in precalculus. I had to look up a very complicated formula to determine how quadratic regression is done mathematically because I never taught it before. It appears that the College Board determined that the good that would be accomplished in showing these techniques and opening the world of all sorts of real-life problems that could be approached using different types of regression was worth using the calculator as a black box. I believe their approach is: regression is on the calculator – so let's use it and learn how to apply it.

  That concept could be argued either way. The math purists feel strongly that math techniques should be taught analytically before technology should be permitted. Still, people very rarely learn how a car actually works at the motor and brakes level. They just know how to drive it. Computer programming was part of the standard curriculum in many schools in the 1970's and 80's, but it is rarely taught today. Every student knows how to use a computer but few know how it works. So using the calculator as a black box is just an aspect of the times to achieve the educational goal of examining real-world applications.

- That goes hand-in-hand with analysis of datasets. In the traditional precalculus course, students might be given an equation that determines, for example, what percentage of reading material students recall as a function of time. That leads students to wonder how this equation was found. Students end up thinking the world works on a series of formulas. In this updated approach by the College Board, students examine data and generate equations that best fit the data. They then use those equations to predict a value in the future or the past and also evaluate the confidence in that answer. And so, the existing precalculus courses have to be amended as that type of problem was never included prior to this new AP precalculus course.

- It has long been my contention that statistics and probability is really the most important math course for students. Most will never use precalculus and calculus but we analyze statistics and make probability judgments on those statistics every day of our lives without even realizing it. Yet, few high schools offer any type of statistics course except AP Stat for the select few. Teaching statistics usually gets relegated to the end of the year in an algebra course, taught by teachers with little concept of it. This new AP precalculus course, while not teaching statistics concepts per se, at least attempts to acknowledge their existence and uses datasets extensively.

- I have always felt that the goal of all math courses should be to apply math to the real world. My teaching always stressed word problems – what we now call modeling. So my precalculus manual was chock full of application problems, especially in the area of exponential growth. So, I have added even more modeling of real-world data to support current College Board policy.

- Teaching these new concepts will be time-consuming and there is limited time in a school year. So certain topics that are in a traditional precalculus course are not mentioned in the new course framework. For instance: Solving quadratic and polynomial inequalities: I believe this topic is a very necessary tool for when we need to test whether functions are increasing or decreasing in certain intervals. It may be taught in an algebra course but I believe it is important enough to teach it in precalculus. And yet there is no mention of it in this new curriculum.

  The entire unit 3, trigonometry, is of concern. Many high schools have a full course in trigonometry as part of their precalculus course. However the College Board makes it clear that proficiency in right angle trig is assumed knowledge. And their curriculum makes no mention of the study of oblique triangles and all the applications that come from that knowledge. I find that curious as trig is probably the first mathematics course in which the real-world applications are important – not asking how old Jack will be in 5 years, the percentage of water in a mixture, or how many cantaloupes Edna bought. The new precalculus trig section is mainly concerned with graphing periodic functions and modeling from that. I have even expanded on what was in the previous manual on this topic. I am not sure why they spend so much time on the graphs of the tangent functions and co-functions as well as their transformations, as I have found few applications for them. So if students have not had a semester course in trigonometry, it is recommended that the entire unit 3 be covered even though there are topics in that unit for which no mention is made in the AP precalculus course framework.

  The same is true with Unit 4.4 on Matrices. There seems to be an assumption that students are familiar with matrices, their use, and how to add or subtract them. Again, if students had no previous formal exposure to matrices, unit 4.4 should be covered in its entirety before moving onto matrix multiplication.

With that in mind, this AP Precalculus manual has topics and even entire chapters that are starred ( * ). This indicates that while the topic is not expressly mentioned in the AP precalculus course framework, I believe it is of sufficient importance to be taught. In the case of the above-mentioned trigonometry, it will be up to the teacher to determine if it is necessary to teach based on the content of previous math courses. It makes no sense in starting the trig section talking about graphing trigonometric functions, as the new curriculum suggests, when students haven't been taught right-angle trig. So although it has been tweaked quite a bit, Unit 3 on trig and polar functions is the least-changed from my precalculus manual, with several starred topics.

In other chapters, the order of teaching concepts that was proposed in the new curriculum I felt needs alteration, For instance, I believe that teaching exponentials and logarithms go hand-in-hand. So very

quickly after I teach exponentials, there is a unit on logarithms. If, for example, we have an example with a cup of coffee cooling and we want to determine how long it will take to get to within 5 degrees of room temperature, doing so with just a knowledge of exponentials requires a graphical solution. With a knowledge of logs, student can then solve exponential equations and get an algebraic answer.

So the order of the AP precalculus course framework is not nearly the same as this AP precalculus manual. Still, every topic in the framework is addressed and on the next pages, the learning objectives from the course framework and the sections in which it appears in this manual are listed. In the solution version, the subtopics will have the AP learning objective stated in the table of contents starting that section. And if that concept is not part of the AP framework, for example, as mentioned above, right-angle trig, it will be so stated.

In this new AP precalculus curriculum, Unit 4: Functions Involving Parameters, Vectors, and Matrices is not assessed on the AP exam and is provided as a courtesy to teachers. It is included in the MasterMathMentor precalculus manual in full detail.

My AP Calculus manuals, both AB and BC, have huge numbers of teachers who use it as their textbook. Teachers like using it as it gives them their daily lesson plans and the exact examples they will show in class. Students like it because they can write in it and they don't have to carry a heavy textbook around, most of which they will never read. Students who graduated my high school frequently told me how they referred to it in college. The format: Classwork … Homework works well. Students who are absent from class know exactly what was covered in class.

Still, these manuals were never meant to be stand-alone teaching tools. A good teacher is required to teach from them. It is my feeling that this new AP precalculus course needs such a teaching guide for instructors because the curriculum is somewhat different to what they are used to. And they certainly want to be absolutely sure that they are teaching exactly the material that will be tested in AP precalculus exam in May.

I have developed this manual over a period of 5 months – from November of 2022 when I first heard about the new AP Precalculus course to March of 2023. In proofreading it, I have found many errors, some math, some typos, some cosmetic and I have corrected them all. Every time I proofread it, I find something else. Feel free to let me know any errors that you have found and I will quickly update it. Contact me at team@mastermathmentor.com.

As with just about all my materials, the student manual is free while the solution version has a cost associated with it. I suggest that you download the free versions to get a sense of the new course before purchasing the solutions. Those who purchase solutions will be provided with links that always point to the most updated solutions.

Once there is information about the AP precalculus exam, I will also be creating sample AP precalculus tests that teachers can download. It is my hope that I eventually will be able to create YouTube videos similar to the ones that I created for AP Calculus. Those have garnered over 100,000 views so it is quite apparent that they are helpful to teachers and students.

If there are any suggestions for this AP Precalculus manual – topics or even problem types, feel free to contact me at team@mastermathmentor.com.

Yours mathematically,
Stu Schwartz and Fanucci the Cat