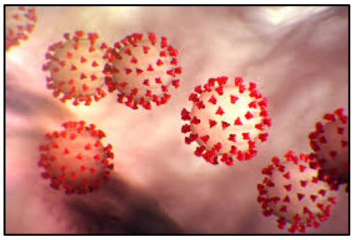
Flattening the Coronavirus Curve Using AP Calculus or AP Statistics

by Stu Schwartz – www.mastermathmentor.com



It is March 18, 2020 and many of us are in some time of quarantine for coronavirus (COVID-19), much of it being self-imposed. The term that is used is "social distancing." And while this makes sense from a personal point of view (the fewer people you contact, the less of a chance you have of catching the virus from someone), there are greater reasons for the good and welfare of the general population.

The term that is bandied about the Internet and the news stations explaining why social distancing is so important is "flattening the curve." It has become a slogan, but I doubt that most people understand what it means mathematically. So, let's look at it.

I thought it would be instructive to devise several simplified math problems based on social distancing because it uses the concept of flattening a normal (bell-shaped) curve. These families of normal curves are the foundation of the AP statistics course, but AP calculus students know little about them and their properties. I wanted some applicable math problems that both students of statistics and calculus can do. So I created a basic short lesson on normal curves for AP calc students, focusing on the equation of the curve (which the AP statistics students never learn). So while both students should be able to do the problems, each will tackle it in a different way. Most will rely on the calculator with the statistics students using the built-in statistics feature to calculate the area under a normal curve while the calculus students will use the calculator's ability to take definite integrals.

Students will find it interesting to see that some are easily solved in statistics are much more difficult in calculus and vice versa.

Calculus students should read pages 2 – 4 to get an overview of normal curves. Statistics students should look at page 5 to give them a refresher on how to use the calculator to work with normal curves. The 10 problems, all based on a certain scenario are given on page 7. Solutions are supplied but try and work out the problems on your own before you look at them, especially problems 1 – 6. Calculus students might also look at page 5 after they solve the problems with calculus to see how easy it is to solve some of these problems using the built-in statistic functions.

Once the nature of normal curves are understood, these problems are rather straightforward (especially problems 1 – 6). For the statistics student, they are both typical of ones that appear on the AP exam. For the calculus student, several give good practice in differentiating complicated expressions. In terms of reality, they certainly are simplified as there are many other variables that control the spread of the virus. Hopefully both will explain to both AP calculus and statistics students why social distancing is so important in these very troubled, uncertain, and downright scary times.

Please stay safe.

- Stu & Fanucci the Cat
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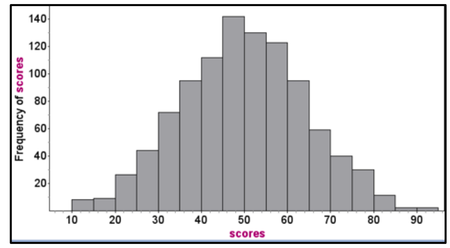
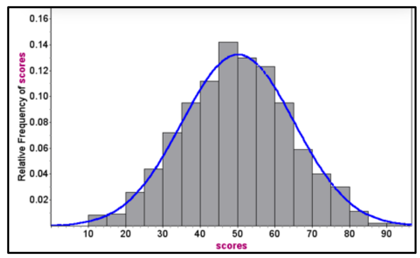
For AP Calculus Students:

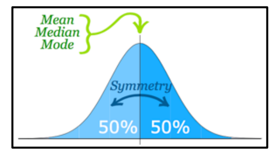
You have not been introduced formally to the normal (bell-shaped) curve, so here is a short lesson. Such curves explain the distribution of many examples of data in the real world. All normal curves show the frequency of data close or near to its mean (average). The further from the mean, the less frequent the data.

Suppose that 1,000 students took a test based on 100 points with scores being integers. Typical results appears like the histogram to the right with most students scoring around the mean of 50 and fewer students scoring much higher or much lower. The scores are placed into bins: [1, 5), [5,10), [10,15), … [95,100]. The y-axis is the frequency of the scores. For instance, there are about 140 scores in the [45,50) bin.

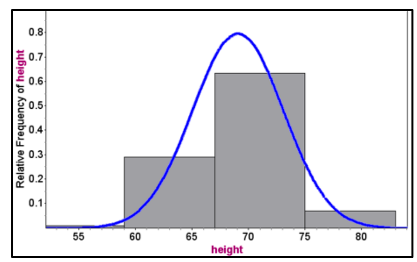


If we change the y-axis into the relative frequency of the scores, then we get the percentage of scores in each bin. For instance the relative frequency of scores in the [45,50) bin is approximately $140/1000 = 0.014 = 1.4\%$. We then fit a curve to the tops of these rectangle bins, and we get close to a normal curve. In a perfect normal curve, we have symmetry at the mean with 50% of the data on either side of the mean. So we would expect half of our scores to be above 50 and half to be lower.

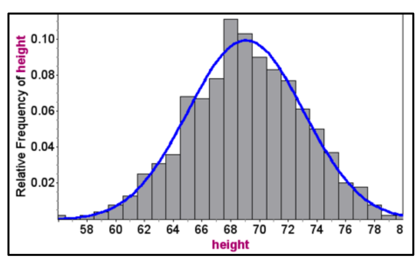


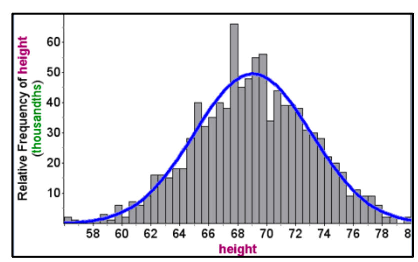


Data like these test scores are discrete because there are a finite number of possible scores. Since scores are from 0 to 100 and are integers, there are 101 possible scores. However, if we were looking at heights of men, the data is continuous in that there are an infinite number of heights. Someone 68 inches tall is taller than someone 67.99 inches tall. We can still create relative frequency histograms, but it is uncertain how many bins to show. Too few bins loses the bell-shaped pattern of the data with the bin width quite large. Too many bins gives too much detail with the bin widths being very narrow and the heights very small.

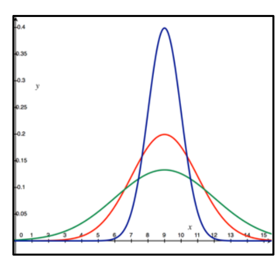


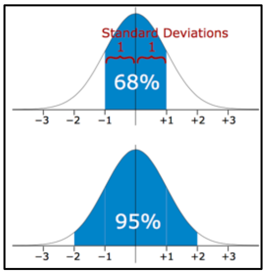
In this histogram of 1,000 men's heights, we see that about 8% of the men have heights between 71.5 and 72.5 inches. The normal blue curve predicts that between 7.5% and 8.5% of the men will have heights between 71.5 and 72.5 inches. If we make our bin width smaller, notice how the blue curve becomes a better predictor of the number of heights within that very small bin. Calculus students should realize how this mirrors your knowledge of Riemann sums.





While the mean gives you the center of the distribution, the standard deviation is a measure of how spread out the data is from the mean. The greater the standard deviation, the greater the spread. In the figure to the right, all 3 normal curves have a mean of 9. But the blue curve has a standard deviation of 1, the red curve a standard deviation of 2, and the green curve, a standard deviation of 3. Of the three curves, the green curve is the most spread out from the mean.





In a perfectly normal distribution, about 68% of data values lie within one standard deviation of the mean and 95% of the data values lie between 2 standard deviations of the mean. 99.7% of the data values lie between 3 standard deviations of the mean. So if you know the mean and standard deviation of a data set and you know it follows a normal pattern (many do), you can sketch a picture of the distribution without actually knowing its mean and standard deviation. While statistics courses explain how to find the standard deviation of a data set, it is not something that calculus students need to know. Many times, the standard deviation is given and if there is a set of data available, the calculator can quickly determine it.

In AP statistics, these normal curves are used so often that they are built into the calculator. But for calculus students, we are interested in representing them symbolically. So if *M* is the mean of data distributed normally and *s* is its standard deviation, we can represent a normal curve using the equation:

$$y=\frac{1}{s\sqrt{2\pi}}e^{-0.5\left[\left(\frac{x-M}{s}\right)\right]^2}$$

So the red curve above with mean $M = 9$ and standard deviation $s = 2$, is given by $y=\frac{1}{2\sqrt{2\pi}}e^{-0.5\left[\left(\frac{x-9}{2}\right)\right]^2}$.

If we want the percentage of data whose values is exactly 9, you might think to just to plug 9 into this expression. It gives you 19.95%. However, this makes no sense. Since the data is continuous, to get values that are exactly, precisely equal to 9 is almost impossible. Since y is measured in terms of $\frac{\text{relative frequency}}{\text{bin width}}$, realize that the $\int y\,dx$ (the area under the curve) will be measured in terms of $\frac{\text{relative frequency}}{\text{bin width}}\cdot\text{bin width} =$ relative frequency. So the relative frequency of normal data between a and b is given by $\int_a^b y\,dx$. Many times in AP type problems, you are given a problem that defines a function in terms of an integral. (Ex: given the function $f(x)$, let $F(t)=\int_0^t f(x)\,dx$). This is a real-life example of such a situation. **This means that the normal curve is best used in terms of the area beneath it.**

- If we want the percentage of data in the red curve that lays below 9, we can then integrate this function from $x = 0$ to $x = 9$. $\int_0^9 \frac{1}{2\sqrt{2\pi}}e^{-0.5\left[\left(\frac{x-9}{2}\right)\right]^2} = 49.9966\%$. The reason we do not get exactly 0.5 is that this curve is asymptotic to the x-axis.

- If we want the percentage of data in the red curve that lays within one standard deviation of the mean, we would calculate $\int_7^{11} \frac{1}{2\sqrt{2\pi}}e^{-0.5\left[\left(\frac{x-9}{2}\right)\right]^2} = 68.2689\%$, corresponding with what was stated at the top of the previous page.

• If we want the average percentage of data that lies within one standard deviation of the mean, we calculate

$$\frac{\displaystyle\int_7^{11} \frac{1}{2\sqrt{2\pi}} e^{-0.5\left[\left(\frac{x-9}{2}\right)\right]^2}}{11-7} = \frac{68.2689\%}{4} = 17.067\%$$

• If we want the percentage of data in the red curve that lays above 2 standard deviations of the mean, we would calculate, $\displaystyle\int_{13}^{\infty} \frac{1}{2\sqrt{2\pi}} e^{-0.5\left[\left(\frac{x-9}{2}\right)\right]^2}$. Not having an infinity button, we can use some large number that is at least 4 standard deviations from the mean. Because of the curve being asymptotic to the *x*-axis, it makes little difference what number is used as the top limit. $\displaystyle\int_{13}^{20} \frac{1}{2\sqrt{2\pi}} e^{-0.5\left[\left(\frac{x-9}{2}\right)\right]^2} = 2.275\%$.

• Since all the data must lie under the normal curve, it follows that $\displaystyle\int_{-\infty}^{\infty} \frac{1}{2\sqrt{2\pi}} e^{-0.5\left[\left(\frac{x-9}{2}\right)\right]^2} = 1 = 100\%$.

Finally, we look at the role of the derivative of the normal curve frequency function which is less important. Let our bin width = 1. Then *y* represents the relative frequency of our data with bin width 1. Its derivative is the instantaneous change in the frequency of our data at any value.

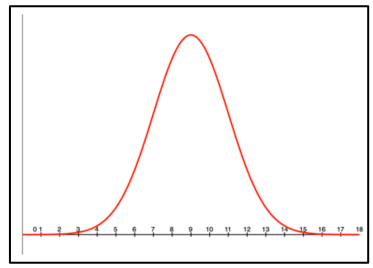
• If we want to know how fast the relative frequency of the data is changing when $x = 9$ and $x = 10$, we find $y'(9)$ and $y'(10)$. Thus $y' = \frac{1}{2\sqrt{2\pi}} e^{-0.5\left[\left(\frac{x-9}{2}\right)\right]^2} \left[\frac{9-x}{2}\right]\left(\frac{1}{2}\right)$ and $y'(9) = 0$. This makes perfect sense as there is a horizontal tangent line at $x = 9$. $y'(10) = \frac{1}{2\sqrt{2\pi}} e^{-0.5\left[\left(\frac{10-9}{2}\right)\right]^2} \left[\frac{9-10}{2}\right]\left(\frac{1}{2}\right) = \frac{-1}{8\sqrt{2\pi}} e^{-1/8} \approx -4.4\%$.

For Statistics Students:

You know all about normal curves and standard deviation which are defined by its mean (μ) and standard deviation (s). So here is a refresher on the use of the calculator. We have 3 calculator statements that are found in the DISTR menu (2nd VARS).

- normalpdf (normal probability density function). This statement is usually only used in graphing mode. To graph a normal curve, use the statement Y = normalpdf (X, μ, s).
- normalcdf (normal cumulative density function). This gives the percentage of data that lies between two values a and b. Its form is Y = normalcdf (a, b, μ, s).
- invNorm (inverse normal). This statement finds the x-value associated with the first p percent of the data. Its form is invNorm (p, μ, s) .

Suppose you have a normal distribution with mean $\mu = 9$ and standard deviation $s = 2$. We know that 95% of the data lies within 2 standard deviations of the mean and 99.7% of the data lies between 3 standard deviations of the mean. So you can sketch the curve to the right but have no idea as to the scale of the y-axis.



- If you want to have the calculator sketch the curve, we use Y1 = normalpdf $(\text{X}, 9, 2)$. Set Xmin to at least 4 standard deviations below the mean and Xmax to at least 4 standard deviations above the mean. Then do a Zoom 0: ZoomFit and you will get the graph with Ymin and Ymax automatically set. Using Xmin = 0 and Xmax = 18, Ymin is just a hair above 0 and Ymax is 0.199.

- If you want to find the percentage of data between 8 and 11, from the home screen use normalcdf (8, 11, 9, 2). You should get 53.28%.

- If you want to find the percent of data above 7, from the home screen use normalcdf(7, large number, 9, 2). The large number should be at least 4 standard deviations above the mean. Theoretically, it is infinity but once you go 4 standard deviations above the mean, it makes little difference. Using normalcdf(7, 99, 9, 2) gives 84.13%.

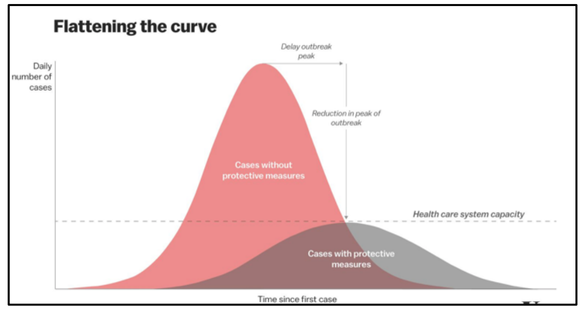
- If you want to find the data value associated with the lowest 25% of the data, use invNorm(0.25, 9, 2). You should get 7.651. So 25% of the data should be lower than 7.651. You can check this using normalcdf(0, 7.651, 9, 2). This will give you 0.24999. It is not exactly 0.25 because the 7.651 value was rounded and the fact that theoretically, the normal distribution stretches to negative infinity. But the difference is negligible.

- If you want to find the data value associated with the top 5% of the data, use invNorm(0.95, 9, 2). You should get 12.290.

For Students Who Have taken Both Calculus and Statistics:

Realize that normalcdf (a, b, μ, s) is equivalent to $\int_a^b y\,dx$ where y is the normal curve describing the relative frequency of the rate of change of the data.

Social Distancing: With the coronavirus spreading quickly in the United States, social distancing is being strongly recommended. The concept is not meant to stop the virus, but to slow its spread. In that way, we don't get a huge spike in the number of new cases with many people getting sick at once. If that were to happen, there wouldn't be enough hospital beds or mechanical ventilators for everyone who needs them, and the U.S. hospital system would be overwhelmed.



The new cases started from someone who was in China or Europe passing it on to someone in the U.S. That person passes it on to someone else and the number of new cases grow to a huge number. After the peak, the virus will still spread but not as fast because people recover and mostly become immune to it. As seen by the graphic, the number of new cases without and with protective measures (social distancing) can be described by normal curves.

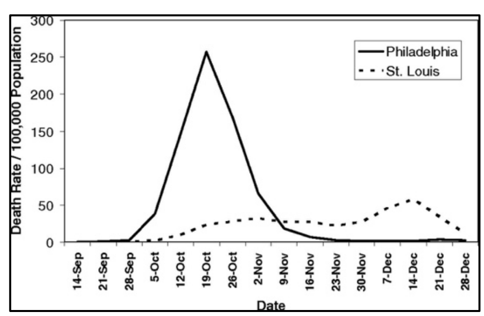
The term "flattening the curve" means that we want to increase the length of time it takes for the virus to infect many people. This might seem counter-intuitive but by doing so and thus increasing the standard deviation of the spread, it allows the health care system to do a better job of keeping up with the great number of cases it will face. It may end up facing relatively the same number of cases but over a greater time period.

Here is a real example. In September 1918, Philadelphia held a planned Liberty Loan Parade to promote the government bonds that were being issued to pay for World War I. But the parade took place when the pandemic commonly called the Spanish flu - the H1N1 virus - arrived in the city of 1.7 million people. The virus swept the world between 1918 and 1919. About a third of the entire world's population -- about 500 million people at the time -- were infected with the virus, and about 50 million died, according to the CDC. Like with COVID-19, there was no vaccine against the virus.

The virus infected 80% of Spain's population. Unlike today with global travel, outbreaks tended to stay in one area. But as US troops came home from World War I, cases popped up and it came to Philadelphia through Navy Yard. In several days, 600 sailors were infected. Yet Philadelphia didn't cancel its Liberty Loan Parade, scheduled for just a little more than a week later. Meant to be a patriotic wartime effort, the parade went on as scheduled on September 28, bringing 200,000 Philadelphians together.

Philadelphia was one of the hardest-hit US cities. More than 12,000 people died in six weeks, with about 47,000 reported cases. By the six-month mark, about 16,000 had died and there were more than half a million cases.

St. Louis canceled its parade while Philadelphia did not. In the end, the death toll in St. Louis did not rise above 700, according to the CDC. "This deadly example shows the benefit of canceling mass gatherings and employing social distancing measures during pandemics," the CDC said.



The graph to the right shows what flattening the curve can mean. The death rate in St. Louis was 1/5 that of Philadelphia at the height of the outbreak. It can also be seen that the outbreak lasted about 2 deadly months in Philadelphia and closer to 3 months in St. Louis. And while there were other factors including a higher Philadelphia population and worse working conditions, the parade was a classic example of what not to do during a pandemic.

So today as more and more cities close restaurants and limit gatherings, it may be useful to look to Philadelphia in 1918. If this event shows us anything, it's that these cancellations can save lives.

The problems:

All the problems below use this basic information:

Suppose that in a country without taking protective measures, the distribution of new cases is a normal curve. Its mean occurs on day 60 (since the first case appeared) with standard deviation 10. A similar country enacts social distancing from the outset and while the mean of the distribution of new cases (also a normal curve) now occurs on day 120 with the standard deviation tripling. Answer these questions for both countries.

Note: questions 1 – 6 are more important than 7 – 10 in terms of flattening the curve and more typical of the everyday questions people ask about the growth of the virus.

1. Sketch the normal curves that describes the new cases. (No calculator needed).

2. Find the calculator equations of the curves describing the number of new cases.

3. If both countries have 1 million people getting the virus, approximately how many fewer people will have gotten the virus 7 days after the new cases per day have peaked if social distancing had been enacted?

4. Compare the average rate of change percentage in new cases for the first half of its growth period and the second half of its growth period.

5. (Statistics only but calculus students should set up an equation that describes this situation.) Approximately how long will it take for 90% of cases to emerge in the two countries described above?

6. Draw graphs that show the cumulative percentage of people infected over time.

7. Find the day when the number of new cases is at a maximum. Calculus students should justify answers.

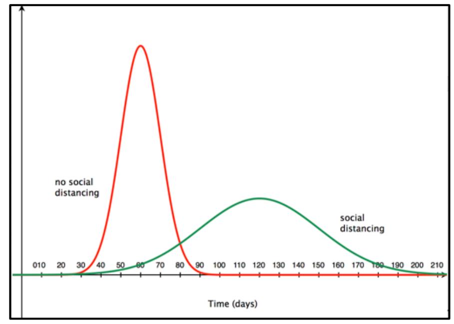
8. Since the normal curve is measured in rate of new cases per day, what is the rate of new cases per day at the time when there are a maximum number of new cases?

9. On what day will the *growth* of new cases per day be at a maximum?

10. (Calculus only) How fast are the new cases per day changing at the time when they are increasing the fastest?

Solutions:

1. Note that in these sketches, we have no idea as to the y-axis scale. In the no social distancing curve, 99.7% of the new cases lie within 3 standard deviations of the mean or from 30 days to 90 days. That is similar to what happened in the U.S. There were relatively few cases for the first month and then it exploded. With social distancing, 99.7% of new cases occurs from day 30 to day 210. We know that the area under the red curve has to equal the area under the green curve (if we assume that social distancing only slows the virus spread). So it cannot go nearly as high.



2.

	Statistics	Calculus
No social distancing	$y = \text{normalpdf}(60,10,\text{X})$	$y = \dfrac{1}{10\sqrt{2\pi}} e^{-0.5\left[\frac{(t-60)}{10}\right]^2}$
Social distancing	$y = \text{normalpdf}(120,30,\text{X})$	$y = \dfrac{1}{30\sqrt{2\pi}} e^{-0.5\left[\frac{(t-120)}{30}\right]^2}$

3.

For stat students:

No social distancing: $\text{normalcdf}(0,67,60,10) = 0.758$

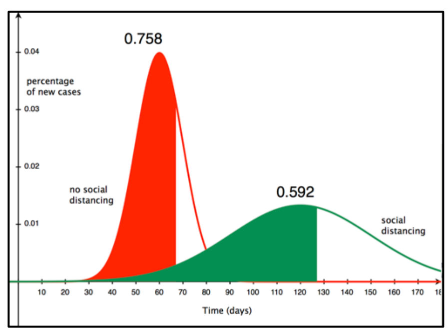
Social distancing: $\text{normalcdf}(0,127,120,30) = 0.592$

$0.758 - 0.592 = 0.166 \qquad 0.166(1000000) = 166{,}000$

For calc students:

D. No social distancing: $\displaystyle\int_0^{67} \frac{1}{10\sqrt{2\pi}} e^{-0.5\left[\frac{(x-60)}{10}\right]^2} dx = 0.758$

Social distancing: $\displaystyle\int_0^{127} \frac{1}{30\sqrt{2\pi}} e^{-0.5\left[\frac{(x-120)}{30}\right]^2} dx = 0.592$



$1000000(0.758 - 0.592) = 166{,}000$

This does not mean that 166,000 fewer people will get the virus. It means that 166,000 fewer people will have gotten it by one week after its peak. So when they do get it the health care system is better equipped to handle them.

4.

Statistics students:

No social distancing: $\dfrac{\text{normalcdf}(0,30,60,10)}{30} = \dfrac{0.0045\%}{\text{day}}$ $\quad \dfrac{\text{normalcdf}(30,60,60,10)}{30} = \dfrac{1.662\%}{\text{day}}$

Social distancing: $\dfrac{\text{normalcdf}(0,60,120,30)}{60} = \dfrac{0.038\%}{\text{day}}$ $\quad \dfrac{\text{normalcdf}(60,120,120,30)}{60} = \dfrac{0.795\%}{\text{day}}$

Calculus students:

No social distancing: $\dfrac{\int_0^{30} \frac{1}{10\sqrt{2\pi}} e^{-0.5\left[\frac{(x-60)}{10}\right]^2} dx}{30} = \dfrac{0.0045\%}{\text{day}}$ $\quad \dfrac{\int_{30}^{60} \frac{1}{10\sqrt{2\pi}} e^{-0.5\left[\frac{(x-60)}{10}\right]^2} dx}{30} = \dfrac{1.662\%}{\text{day}}$

Social distancing: $\dfrac{\int_0^{60} \frac{1}{30\sqrt{2\pi}} e^{-0.5\left[\frac{(x-120)}{30}\right]^2} dx}{60} = \dfrac{0.038\%}{\text{day}}$ $\quad \dfrac{\int_{60}^{120} \frac{1}{30\sqrt{2\pi}} e^{-0.5\left[\frac{(x-120)}{30}\right]^2} dx}{30} = \dfrac{0.795\%}{\text{day}}$

These numbers may appear small but in our countries of 1 million people, the average change is:

	First half	Second half
No social distancing	$\frac{\text{45 new cases}}{\text{day}}$	$\frac{\text{16,620 new cases}}{\text{day}}$
Social distancing	$\frac{\text{380 new cases}}{\text{day}}$	$\frac{\text{7,950 new cases}}{\text{day}}$

5.

Statistics students:

no social distancing: $\text{invNorm}(0.9,60,10) \approx 73$ days

social distancing: $\text{invNorm}(0.9,120,30) \approx 158$ days

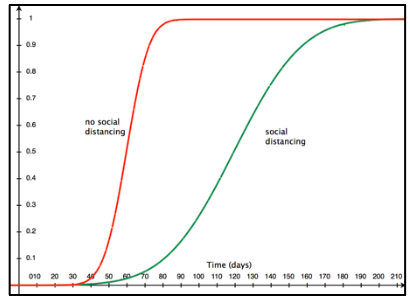
The price we pay for flattening the curve is that the virus will be with us for over twice the length of time unless other factors slow it down.

Calculus students:

We would have to solve the equations $\int_0^{k} \frac{1}{10\sqrt{2\pi}} e^{-0.5\left[\frac{(t-60)}{10}\right]^2} dt = 0.9$ and $\int_0^{k} \frac{1}{30\sqrt{2\pi}} e^{-0.5\left[\frac{(t-120)}{30}\right]^2} dt = 0.9$

Since we cannot integrate these expressions, the only way to do this is by trial and error.

6. This can be done on the calculator. It is easy for statistics students: The graphs are Y1 = normalcdf(0,X,60,10) and Y2 = normalcdf(0,X,120,30). They each will take about 10 seconds to generate. Look at the figure to the right to see the best scale. For calculus students:



Let $Y1 = \dfrac{1}{10\sqrt{2\pi}} e^{-0.5\left[\frac{(t-60)}{10}\right]^2}$ and Y2 = fnInt(Y1,X,0,X).

Let $Y3 = \dfrac{1}{30\sqrt{2\pi}} e^{-0.5\left[\frac{(t-120)}{30}\right]^2}$ and Y4 = fnInt(Y3,X,0,X).

These will be painfully slow to generate as the calculator must use Riemann sum techniques to compute the area under the normal curve for each value of X. Be patient. The graphs are called logistic curves which are taught in the BC calculus course. My blog on the start of the coronavirus discusses these curves.
http://www.mastermathmentor.com/mmm-archive/CoronaVirus.pdf

7.

Statistics Students:

It is obvious that due to the nature of the curves, the maximum occurs at at the mean values.

No social distancing: Day 60 Social distancing: Day 120

Calculus Students:

No social distancing: $y' = \dfrac{1}{10\sqrt{2\pi}} e^{-0.5\left[\frac{(t-60)}{10}\right]^2} \cdot \dfrac{-(t-60)}{100} = 0 \Rightarrow t = 60$

Social distancing: $y' = \dfrac{1}{30\sqrt{2\pi}} e^{-0.5\left[\frac{(t-120)}{30}\right]^2} \cdot \dfrac{-(t-120)}{900} = 0 \Rightarrow t = 120$

8.

Statistics students:

No social distancing: $\text{normalpdf}(60,60,10) = 3.989\%/\text{day}$

Social distancing: $\text{normalpdf}(120,120,30) = 1.330\%/\text{day}$

Calculus students:

No social distancing: $y(60) = \dfrac{1}{10\sqrt{2\pi}} e^{-0.5\left[\frac{(60-60)}{10}\right]^2} = \dfrac{1}{10\sqrt{2\pi}} e^0 = \dfrac{1}{10\sqrt{2\pi}} = 3.989\%/\text{day}$

Social distancing: $y(120) = \dfrac{1}{30\sqrt{2\pi}} e^{-0.5\left[\frac{(120-120)}{30}\right]^2} = \dfrac{1}{30\sqrt{2\pi}} e^0 = \dfrac{1}{30\sqrt{2\pi}} = 1.330\%/\text{day}$

It can be seen that since the maximum value of new cases occurs at the mean value, increasing the standard deviation by a factor of k will reduce the percentage of maximum new cases per day by a factor of k.

9.

<u>Statistics students</u>:

This is easy if they know that the maximum growth rate occurs one standard deviation to the left of the mean:

No social distancing: Day 50 Social distancing: Day 90

<u>Calculus students</u>:

If the number of new cases per day is y, the growth rate is y' and to maximize the growth rate, set $y'' = 0$

No social distancing:

$$y' = \frac{1}{10\sqrt{2\pi}} e^{-0.5\left[\frac{(t-60)}{10}\right]^2} \left(\frac{60-t}{100}\right) \Rightarrow y'' = \frac{1}{10\sqrt{2\pi}} e^{-0.5\left[\frac{(t-60)}{10}\right]^2} \left(-\frac{1}{100}\right) + \left(\frac{60-t}{100}\right)\left[\frac{1}{10\sqrt{2\pi}} e^{-0.5\left[\frac{(t-60)}{10}\right]^2} \left(\frac{60-t}{100}\right)\right]$$

$$y'' = \frac{1}{10\sqrt{2\pi}} e^{-0.5\left[\frac{(t-60)}{10}\right]^2} \left[-\frac{1}{10} + \frac{(60-t)^2}{100^2}\right] = 0 \Rightarrow \frac{1}{10} = \frac{(60-t)^2}{100^2} \Rightarrow 100^2 = 100(60-t)^2 \Rightarrow 60 - t = \pm 10$$

$x = 50, x = 70$: New cases per day are increasing the fastest at $x = 50$ because $y'(50) > 0$

Social distancing:

$$y' = \frac{1}{30\sqrt{2\pi}} e^{-0.5\left[\frac{(t-120)}{30}\right]^2} \left(\frac{120-t}{900}\right) \Rightarrow y'' = \frac{1}{30\sqrt{2\pi}} e^{-0.5\left[\frac{(t-120)}{30}\right]^2} \left(-\frac{1}{900}\right) + \left(\frac{120-t}{900}\right)\left[\frac{1}{30\sqrt{2\pi}} e^{-0.5\left[\frac{(t-120)}{30}\right]^2} \left(\frac{120-t}{900}\right)\right]$$

$$y'' = \frac{1}{30\sqrt{2\pi}} e^{-0.5\left[\frac{(t-120)}{30}\right]^2} \left[-\frac{1}{900} + \frac{(120-t)^2}{900^2}\right] = 0 \Rightarrow \frac{1}{900} = \frac{(120-t)^2}{900^2} \Rightarrow 900^2 = 900(120-t)^2 \Rightarrow 120 - t = \pm 30$$

$t = 90, t = 120$: New cases per day are increasing the fastest at $t = 90$ because $y'(90) > 0$

10.

<u>Calculus Students</u>:

No social distancing: $y'(50) = \frac{1}{10\sqrt{2\pi}} e^{-0.5\left[\frac{(50-60)}{10}\right]^2} \cdot \frac{-(50-60)}{100} = \frac{1}{10\sqrt{2\pi}} e^{-0.5}\left(\frac{1}{10}\right) = \frac{0.242\%}{\text{day}^2}$

Social distancing: $y'(90) = \frac{1}{30\sqrt{2\pi}} e^{-0.5\left[\frac{(90-120)}{30}\right]^2} \cdot \frac{-(90-120)}{900} = \frac{1}{30\sqrt{2\pi}} e^{-0.5}\left(\frac{1}{30}\right) = \frac{0.027\%}{\text{day}^2}$

These may not seem big, but in a population of 1 million, the number of cases a day increase by:

no social distance: 2,420 cases a day per day social distancing: 270 cases a day per day

Comments:

Some of these answers are somewhat terrifying. # 8 states that with no social distancing, the number of new cases increases on the average of 16,620 new cases a day at the height of the virus' spread. # 9 states that with social distancing, we might have to quarantine ourselves for about a half year. Remember that these are just examples with the numbers not being the truth as what the current situation is. The non-social distancing country with a distribution mean of 60 days is close to what China experienced. But the standard deviation of 10 days is made up. The second country with a mean of 120 days and tripling the standard deviation is completely made up to make the numbers easy to work with. **It is not a model of the situation we are facing**.

Again, these problems are based solely on the mathematics that assumes that all social distancing will do is to slow the spread of the disease and no other factors. Consider that a person with COVID-19 and hasn't been tested for it might infect on average of 1 person every 2 days. In one week, that means that 8 people could be infected. If that person had quarantined himself, that means that 8 fewer people would get the disease. Do the math. 1 million people quarantine themselves and that means 8 million fewer cases. Of course, that is simplified as well. But there is no doubt that social distancing can make a difference. So please, just resist the temptation to be with others. You can talk on the phone all you want and test and use Skype. The more we social distance, the better things will be for all of us.

A terrific simulation of the impact of social distancing can be found at:

https://www.washingtonpost.com/graphics/2020/world/corona-simulator/?fbclid=IwAR1uHBanh8XaPw3lqNriz0ACNJ_V2aHS8urzbV4eI9p8swAdNlD579u9dDI

Students of AP calculus or AP statistics, your teacher might be giving you some work to do at home. If you need some help, email me at team@mastermathmentor.com, explain the problem or concept with which you are having problems, and I will get back to you with an explanation. Just wash your hands first!

Teachers, the same offer goes to you. If you need some tip on teaching a concept online and how to use some of my materials, email me at team@mastermathmentor.com, and I can point you to the appropriate pages in the wealth of free materials on www.mastermathmentor.com. There is some talk of making the AP exam online (https://www.latimes.com/california/story/2020-03-17/coronavirus-ap-tests-college-board) although I cannot conceive of how this can occur. Still, it is prudent to plan for different circumstances and to keep the learning going. If there is anything I can do to help, let me know.

I hope this material has helped you to understand the mathematics behind this terrible virus and more importantly, how we can mitigate the impact. Please stay safe, practice social distancing skills, and remember to smile!

- Stu