

CALCULUS
CRUISING
MasterMathMentor



SOLUTION VERSION

This material was developed by Stu Schwartz for an Advanced Placement™ AB or BC Calculus class as a review for the AP Calculus exam. It is not endorsed by the College Board.

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Thanks to my friend and business partner Ted Tyree for his unwavering support and creativity in all my projects and for always finding time to bring my materials free to anyone who wants them through the Internet. Thanks to Sam Tsui (Yale University, 2011) for developing the www.mastermathmentor.com logo below (it is one cat ago but it is original to the website and I am still loyal to it).

A big shoutout to Jason Nocera of Niche Cartoons <http://www.nichecartoons.com> for reproducing the iconic scene from “Titanic” in the cover art starring Fanucci the Cat. I wish I were still that thin.

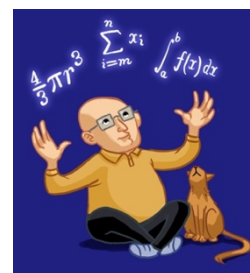
Thanks also to Dr. Sheldon Goldberg for using his cruise experience to make sure that the problems made sense.

Most of all, thanks to the wonderful crew of Caribbean Princess who patiently answered all of my questions, many on topics that they rarely think about. It was a joy creating this set of problems in that wonderful environment.

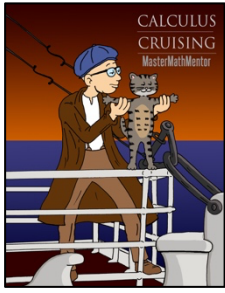
In a project of this nature, it is extremely possible that mathematical typos have worked their way into this manual. It is terribly difficult to find errors in the editing process. So, if you do find mistakes of any type, in the student or solution versions, please let me know at team@mastermathmentor.com. I will immediately correct them, and update the revision to our server. You will always have access to the most recent and accurate version.

As always, questions or comments are welcome. Best regards,

Stu Schwartz
June, 2023



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CALCULUS CRUISING

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Topics are the primary concepts tested in these problems

C – denotes all or part of the problem require use of graphing calculator

BC – denotes all or parts of the problem contains BC curriculum

IRC – Indicates that a vital concept in the problems involves integrating a rate of change

Introduction to Calculus Cruising

Calculus Topics and Associated Problems

Welcome Aboard

	IRC	Topic 1	Topic 2	Topic 3
1. ^C Security	*	Average Rate of Change	Absolute Max/Min	
2. Embarkation		Differential Equations	Integration Techniques	
3. ^C . Getting Your Luggage	*	Differentiation Techniques	Absolute Max/Min	

Getting Around the Ship

4. The Elevators	*	Straight-Line Motion	Integration Techniques	
5. ^C The Promenade Deck		Absolute Max/Min	Continuity	Differentiation Techniques
6. ^C Opening Doors		Related Rates	Absolute Max/Min	Differentiation Techniques

Staterooms

7. ^C Servicing the Room	*	Function Analysis	Average Value	Integration Techniques
8. ^C Bathroom Water		Continuity	Average Rate of Change	Definite Integral as Area

Guest Services

9. ^C . The Purser's Desk		Average Rate of Change	Riemann Sums	Mean-Value Theorem
10. Supplying Pool Towels		Riemann Sums	Definite Integral as Area	Absolute Max/Min
11. ^C . Internet		Related Rates	Differentiation Techniques	Implicit Differentiation
12. Medical Facility (BC)		Differential Equations	Euler's Method	Logistic Growth

Dining

13. Knives and Forks	*	Definite Integral as Area	Function Analysis	Accumulation Function
14. Dining Room Shapes		Area	Volume	
15. ^C . Lobster Night (BC)		Function Analysis	Improper Integrals	2 nd Fundamental Theorem
16. Buffet Bacon	*	Definite Integral as Area	Accumulation Function	Absolute Max/Min

The Lido Deck

17 ^C . Lounging on Deck	*	Riemann Sums	Average Rate of Change	Absolute Max/Min
18 ^C . Fitness Center	*	Function Analysis	Differentiation Techniques	
19 ^C . The Terrace Pool	*	Related Rates	Intermediate Value Thm	Differentiation Techniques
20 ^C . Deck Party (BC)		Polar Equations	Average Value	Function Analysis
21. The Casino (BC)		Series Convergence	Limits	Geometric Series
22 ^C . The Neptune Pool		Area	Volume	Arc Length
23. The Hot Tubs		Riemann Sums	Volume	Related Rates

Snacks and Drinks

	IRC	Topic 1	Topic 2	Topic 3
24 ^C . Drink Package (BC)	*	Accumulation Function	Integration Techniques	Integration by Parts
25 ^C . Ordering Beverages	*	Integration Techniques	Function Analysis	
26 ^C . Pizza!		Average Rate of Change	Derivative Techniques	Linear Approximation
27 ^C . Ice Cream		Linear Approximation	Function Analysis	Differential Equations
28 ^C . Int'l Café (BC)	*	Taylor Polynomials	Average Rate of Change	
29 ^C . The Salty Dog!		Differential Equations	Integration Techniques	Limits
30. Mixing Drinks		Average Value	Volume	Related Rates

Shore Excursions

31. ^C Tendering (BC)		Differential Equations	Integration Techniques	Logistic Growth
32. Getting to Town		Riemann Rums	Rolle's Theorem	Straight-Line Motion
33. ^C Gangway!		Related Rates	Differentiation Techniques	Implicit Differentiation

Entertainment

34 ^C . Lounge Acts	*	Average Rate of Change	Accumulation Function	
35 ^C . Movies Under Stars		Related Rates	Inverse Trig	Absolute Max/Min
36 ^C . Broadway Revue	*	Absolute Max/Min	Average Value	

Ship Movement

37 ^C . Paralleling Other Ship		Straight-Line Motion	Integral Techniques	Differentiation Techniques
38 ^C . Port Navigation (BC)		Vector-Valued Functions	Straight Line Motion	Differentiation Techniques

Sad to Leave

39. Settling the Bill		Slope-Fields	Differential Equations	Linear Approximation
40 ^C . Debarkation	*	Absolute Max/Min		

As is true in the actual AP exam, each problem is worth 9 points and a general scoring guideline is given for each sub-question.

Introduction to Calculus Cruising

The idea for Calculus Cruising came some months ago when I was flipping through channels and I came across that iconic scene in “Titanic” when Jack brings Rose to the front of the ship to experience being “King of the World.” I thought how neat it would be to have the cover of one of my [MasterMathMentor.com](https://www.mastermathmentor.com) projects to have pretty much that scene except it would be my cat Fanucci feeling the wind in his face. So I had the cover concept and all I needed was a set of calculus problems to follow it!

I began to think of how many situations aboard a cruise ship lend themselves to the types of questions that are frequently asked on the AP Calculus exam. I call them “Give and Take” problems. People take towels and they are replaced. People take bacon from the buffet and the ship’s galley crew brings more. People use utensils and they are washed and returned for use. There are countless examples. Still, in order to find enough examples of calculus on a cruise ship, I believed I needed to be in that ship environment and see it and feel it firsthand.

So I booked a weeklong cruise on the Caribbean Princess out of Ft. Lauderdale to the Eastern Caribbean on May 5, 2023. On the cruise, I had my laptop and TI calculator with me all the time – the dining rooms, buffet, pools, theatre, bathrooms, and even shore excursions. I was looking for stimulus and pretty much found it wherever I went. I had a goal of writing 12 complete free response problems during the cruise. If I could do that, I would complete the work when I returned home and have Jason Nocera from Niche Cartoons create a cartoon.



However the ground rules were that the problems had to make sense and numbers used needed to be somewhat accurate to the ship demographics. Many problems give rates of change: how fast people eat bacon, how fast a lounge fills, the speed that towels are washed. And obviously, the integral of that rate of change will give how many strips of bacon are eaten, how many people can fit in the lounge, and how many towels are returned for use. It was important to me that these numbers made sense for the Caribbean Princess.

In creating these problems, I tried to use the material that is typically tested in the AP exam – polynomial, trig, logarithmic, and exponential functions that are introduced in the stem of these problems. Most of the formulas though are made up and have not been researched and they are there simply to test student’s ability to differentiate and integrate them. But the answers they provide do make sense for a cruise ship.

There are some major differences between what students see in the free response section of the AP exam and the problems introduced in **Calculus Cruising**. Here are some of them.

In the AP Exam, problems can be purely numeric and theoretical. You are given a graph of $f'(t)$ over some domain and asked questions about $f(t)$.

There is no attempt to apply this problem to the real world. The units of t are not even mentioned.

In the AP exam, functions are defined over some domain.

“A rate of change $R(t)$ might be given for $0 \leq t \leq 8$

and a student is asked to find $\int_0^8 R(t) dt$.”

In **Calculus Cruising**, the graph of $f'(t)$ might represent the rate that people take a certain type of cake from the buffet in slices per minute. Students would be expected to know what $f(t)$ means in that context.

In **Calculus Cruising**, a rate of change might be given between two times of the day.

“Passengers enter a lounge at a certain rate function R for 30 minutes starting at 8:00 PM. A question might be asked as to how many people are in the lounge when the show begins at 8:30. It is up to the students to realize that rate is a function of time t for $0 \leq t \leq 30$.”

In typical AP free response questions, two competing processes take place over a given time period.

“Water is filling a tank at rate $f(t)$ gallons/min and a hole in the tank is causing it drain at a rate of $d(t)$ gallons per minute where $0 \leq t \leq 15$. How much water is in the tank at $t = 15$?”

In the actual AP exam, 2 problems are designated as Calculator Active while 4 problems do not require or allow a calculator. Those 4 problems focus more on technique than actual numbers. There are a lot of “Show that...”, “Explain why...”, and “Justify your answers” than in the calculator-based problems. And when numbers are used, they tend to be “nice” – meaning integers or simple fractions.

In the actual AP exam, you are frequently told that functions are differentiable. That opens the door to taking derivatives and integrals. If functions are not differentiable, there isn’t much you can do with calculus.

Calculus Cruising’s compendium of 40 problems covers most but not all topics addressed in the AP exam, both AB and BC. There are 15 problems whose main focus is an integral of a rate of change. For instance, you are told that the rate that people line up to get off the ship is $R(t)$ and of interest is the number of people in line

over a time period which is $\int_{t_1}^{t_2} R(t) dt$ or the average number of people in line: $\int_{t_1}^{t_2} R(t) dt / (t_2 - t_1)$. This is an

area that students typically struggle with and the myriad of such examples will allow students to become experts at them because they always show up in a free-response question on the AP exam, whether it is in a real-world context or not.

In **Calculus Cruising**, the time periods may not be the same.

“Bags are loaded on the ship at the rate of $l(t)$ bags per hour from 10 AM to 3 PM. Bags are brought to the stateroom at the rate of $s(t)$ bags per hour from 12 noon to 6 PM. How many bags still need to be delivered to rooms at 6 PM?”

Students would need to set up the two functions with function l from $t = 0$ to $t = 5$ and function s from $t = 2$ to $t = 7$.

In **Calculus Cruising**, 28 of the 40 problems require calculators. All of these problems focus on real life situations and answers are rarely “nice”. In the real world of cruising, a cabin attendant doesn’t take exactly 20 minutes to clean a room, people don’t get sick at the constant rate of 10 people a day and the ship doesn’t have exactly 1,000 deck chairs. Expect fractions with strange denominators and ugly decimals. That makes the problems realistic and not contrived. And obviously more difficult.

In **Calculus Cruising**, most problems tend to deal with rates that passengers perform some action or the number of passengers who fit some category. You cannot have a fraction of a passenger. So the functions, with a range of positive integers, are not really continuous. Still, when the number of passengers is large, these functions will appear to have the same characteristics of differentiable functions and that is why we can use them for good approximations. We make that clear by stating that the functions are models.

Try as I might to include them, there were AP Calculus topics for which it was difficult to find a real-world application. There just isn't much application for proving differentiability or l'Hospital's Rule or the 2nd derivative test. And while BC topics were even more challenging to apply (there are 12 problems that use BC concepts), I was quite pleased to incorporate a problem requiring Taylor Polynomials, and even amazed myself by managing a problem requiring Infinite Series. Still, although just about every AP calculus topic is represented in this set of 40 problems, some may only be mentioned once and as a sub-question. So this set of problems is somewhat skewed to a certain type of problem situation.

I did not create all of these problems completely from scratch. I had with me all of the free response questions asked in the AP exam going back to the 90's. I used them for inspiration and I tried to build real-life problems based on the types of questions that they asked. But all of the problems in this collection are original. On the next page, there is a table showing important calculus concepts and the problems in which they are tested.

With all of this in mind, I would not recommend using a group of these problems as a practice AP exam that is graded. A good practice AP exam will include problems that are strictly numerical, not having real-world applications. Also be aware some of these problems could take students longer than the expected 15 minutes usually allotted per problem. Since use of the calculator is stressed, just punching in a complicated function to be integrated takes time. I believe the best way to use this problem set is a review or even as an exercise after the AP exam where time is not a factor. The AB problems would be a great summer project for students who are entering BC calculus. And unlike some of my other offerings on MasterMathMentor.com like **Diving Into AP Calculus**, these problems shouldn't be tackled until the entire course has been covered due to the global nature of the problems relative to the AP curriculum.

Many students today will have gone on cruises and hopefully they will find these problems interesting and realistic. It is my hope that students who have not had a cruise experience will be able to do so in order to see how a small, closed-in world works. And for those who have cruised, I would like them to appreciate the issues that go into running a city at sea.

It was great fun creating these problems while on the ship although the sight of me doing calculus among the pool parties, karaoke, gourmet meals, and entertainment must have been amusing to some (thankfully I did not have to explain myself when the on-stage comedian asked me why I brought a calculator to the show). Several passengers asked what I was doing and shook their heads in wonderment when I told them and some even said that there is a rule that you cannot do work on a cruise!

I am extremely grateful to the crew of Caribbean Princess who went out of their way to help me to make the problems of **Calculus Cruising** realistic.

For my next cruise, I am thinking about showing the use of AP Statistics on cruise ships which I know will be considerable. Stay tuned.

So I hope you enjoy the 40 real-world problems of **Calculus Cruising** and marvel how calculus is all around us if we search for it. And Fanucci the Cat gets to be King of the World!

Stu Schwartz
MasterMathMentor.com



Calculus Topics and Associated Problems

In **Calculus Cruising**, a major concept is finding the integral of a rate of change. 15 of the 40 problems ask this question in one way or another. The problems that do so are:

Integral of a rate of change: 1, 3, 4, 7, 13, 16, 17, 18, 19, 24, 25, 28, 34, 36, 40

The tables below give important calculus concepts and the problems that stress them.

AB Topic	Problems testing this concept									
Limits	21	29								
Average Rates of Change	1	8	9	17	26	18	34			
Differentiation Techniques	3	5	6	11	18	19	26	33	37	38
Linear Approximation	26	27	39							
Implicit Differentiation	11	33								
Continuity	5	8								
Intermediate Value & Rolles Thm	9	19	32							
Related Rates	6	11	19	23	30	33	35			
Function Analysis	7	13	15	18	20	25	27			
Absolute Max/Min	1	3	5	6	10	16	17	35	36	40
Integration Techniques	2	5	7	24	25	29	31	37		
Definite Integral as Area	8	10	13	16						
Riemann sums	9	10	17	23	32					
Accumulation Function	13	16	24	34						
Average Value/2nd FTC	7	15	20	30	36					
Straight-Line Motion	4	32	37	38						
Inverse Trig	35									
Area (Geometric)	14	22								
Volume	14	22	23	30						
Slope Fields	39									
Differential Equations	2	12	27	29	31	39				

BC Topic **Problems testing this concept**

Logistic Growth	12	31
Improper Integrals	15	
Polar Equations	20	
Series Convergence	21	
Euler's Method	12	

BC Topic **Problem testing this concept**

Geometric Series	21
Arc Length	22
Integration by Parts	24
Taylor Polynomials	28
Vector-Valued Functions	38