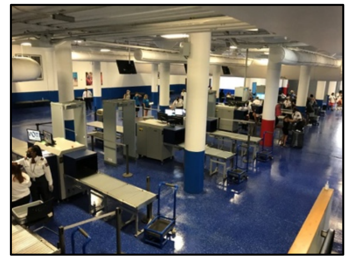
1. (Calculator) The first step to embarking on a cruise is to go through security. It is similar to airline security. There are many people boarding so there are a number of security lines. The longest lines are early in the day before 11:00 AM when boarding begins and the security lines open. For Caribbean Princess, all 3,140 passengers will have to go through security.



The rate that passengers are processed through security at embarkation for the first 30 minutes is given in the table below, 5 minutes apart. However, the data for 11:20 AM is missing.

time	11:00	11:05	11:10	11:15	11:25	11:30
$r(t)$ (passengers/min)	25	16	16	20.5	25	16

(a) What is an approximation for the average rate of change of r at 11:20 AM? Specify units. **(1)**

(b) An approximation to r is given by $r(t) = -0.006t^3 + 0.27t^2 - 3t + 25$ where t is measured in minutes after 11:00 AM, $0 \le t \le 30$. Determine the approximation to the instantaneous rate of change of r at 11:20 AM. Specify units. **(3)**

(c) Determine the time when this rate r is at a maximum. Justify your answer. **(3)**

(d) How many people have gone through security by 11:30 AM? **(1)**

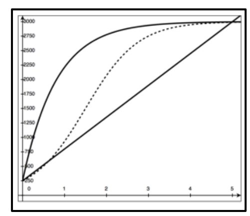
(e) Let $p(t)$ represent the rate that passengers go through security after 11:30 AM and $t = k$ be the time when all 3,140 passengers have gone through security. Write but do not solve an equation determining the value of k. **(1)**

2. A model for the rate that people embark the Caribbean Princess in people per hour is proportional to the difference between the guest capacity of the ship (3,140) and the current number of people P onboard with the coefficient of proportionality equaling $\frac{5}{4}$ and t representing time in hours. At 11 AM, when new guests are allowed aboard, there are already 240 passengers aboard the ship who are continuing their previous cruise.

(a) Write the differential equation and initial condition describing embarkation. **(1)**

(b) Is the number of passengers onboard increasing faster when 1,000 people are aboard or 2,000? Explain your reasoning. **(1)**

(c) Find $\frac{dP^2}{dt^2}$ in terms of P. Use $\frac{dP^2}{dt^2}$ to explain why the graph of P could only resemble one of the following. **(2)**



(d) Use separation of variables to find $P(t)$. If the last passengers board at 4:00 PM, how many passengers are on the cruise? **(5)**

3. (Calculator) Embarkation has become a relatively simple process but it can take hours for your luggage to find your room. Starting when people tend to start checking in at 10 AM, bags are given to porters who take them inside the terminal and are then, after going through their own security (looking for weapons and liquor, mostly), are brought to a cargo hold inside the ship. They are separated as to the deck of the stateroom, identified by a color-coded tag. At 12 noon, the arduous process of crew taking them up to the staterooms begins and lasts all afternoon and even into the evening. So there is no advantage in getting to the ship early so that you will have your bags right away.

On the Caribbean Princess, the rate, measured in bags per hours, at which bags arrive at the terminal is modeled by $R(t) = 300 + 315\cos\left(\dfrac{t^2}{20}\right)$ where t is measured in hours after 10:00 AM and $0 \le t \le 6.5$. At 12 noon, there are 200 bags already on the ship (from passengers who had them shipped) and the ship's crew transports the bags to the rooms at the rate of 575 bags per hour until 5:30 PM.

(a) Find $R'(3)$. Using correct units, interpret your answer in the context of the problem. **(2)**

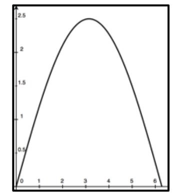
(b) Find the total number of bags that are placed on the Caribbean Princess. **(2)**

(c) Is the number of bags undelivered to staterooms increasing or decreasing at 3 PM? Show the work that leads to your answer? **(2)**

(d) What is the maximum number of undelivered bags from 10 AM through 5:30 PM? Justify your answer. **(3)**

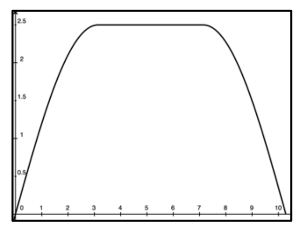
4. On the Caribbean Princess, the height distance between decks is 10 feet. There are 14 elevators and they are vital in getting around the ship. Having one stop at your floor during busy times is always an issue and if it stops at every floor (kids left to roam unattended will frequently hit every floor button), it can take a great deal of time. On this ship, most elevators travel between deck 5 and deck 15. However for superstitious reasons, there is no deck 13, just a deck 12 and then a deck 14.

(a) If an elevator travels from one floor to the one directly above it, its velocity is given by $v(t) = 2.5\sin(0.5t), 0 \le t \le 2\pi$, where t is measured in seconds and v is measured in ft/sec, as shown in the figure to the right. Confirm that the distance between consecutive decks is 10 feet. Show how you get your answer. **(2)**



(b) If an elevator starts at the 5th floor and stops at every floor and it takes 10 seconds for the elevator to open its door and then close its doors on all floors, how much time does it take from the time the doors first close on deck 5 and first open on deck 15? **(2)**

(c) If the elevator skips a deck, it can go faster. For instance, suppose it goes from deck 5 to deck 7. As shown in the figure to the right, It accelerates for the first half floor. Its speed is then constant for 4 seconds and then it decelerates for the next half floor. The deceleration stage is just a mirror image of the acceleration stage. The piecewise function that describes its speed is shown below. Confirm that the distance between two decks is 20 feet. **(3)**

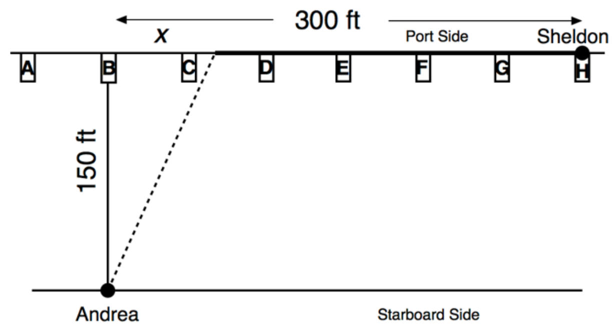


$$v(t) = \begin{cases} 2.5\sin(0.5t), t \le \pi \\ 2.5, \pi < t \le \pi + 4 \\ 5\cos(0.5(t - (\pi - 4))), \pi + 4 < x \le 2\pi + 4 \end{cases}$$

(d) An elevator starting on the 5th floor and going nonstop to the 15th floor has the same acceleration and deceleration as above and its speed is a constant at 2.5 ft/sec for the rest of the trip. How long does it take to travel from deck 5 to deck 15? **(2)**

5. (Calculator) People go on cruises to relax but people are still in a hurry when on a ship. They are always interested in getting to someplace quickly and a ship over 1,000 feet long affords many pathways. Passengers are always deciding what is the most expedient route to take, what elevators are best, and what areas to avoid in getting from one area of the ship to another. Unlike many of the newer ships, Caribbean Princess has a Promenade Deck on deck 7 that allows passengers to walk around the entire perimeter of the ship without crowds.

Suppose Andrea is on the Promenade Deck on the starboard side of the ship and Sheldon is 300 feet away on the port side as shown in the figure to the right. Sheldon texts Andrea to meet him and Andrea must decide on the path to take. She can walk along the wide 150 ft long corridor to door B at 3.4 ft/sec and also along the Promenade deck at 5 ft/ sec. She can also angle in at a distance x, from the corridor, walking through the ship, at a slower 3 ft/sec. (Let us assume that this is possible and there are no stairwells or walls or other impediments along the path). Her goal will be to use the door (A – H) that will minimize her time.



(a) Find the time T it would take Andrea to walk across the corridor and then down the promenade deck to Sheldon. **(1)**

(b) If she angles in, write an expression for the total time T it would take Andrea to walk to Sheldon as a function of x where $0 < x \leq 300$. **(2)**

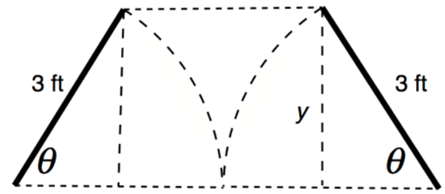
(c) Explain mathematically and in the context of the problem situation why T is not continuous at $x = 0$. **(2)**

(d) Find the minimum value for T. Justify your answer. With that in mind, what door should Andrea use, assuming that the doors are equally spaced? **(4)**

6. 

On cruise ships, heavy doors to outside decks usually open automatically by walking in front of them. This is done so that in case of high winds or inclement weather, the crew can shut off power and so no one can get on deck in dangerous conditions.

The doors to the Promenade deck on Caribbean Princess are 3 feet wide. When they are opened from the inside, they swing open as shown in the figure to the right. Looking down as shown, airspace is created on the deck: 2 triangular sections and a rectangular section. The doors open at a constant rate in 2 seconds and θ is the angle created by the doors and the threshold.



(a) Find $\dfrac{d\theta}{dt}$. **(1)**

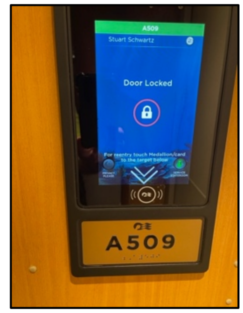
(b) Write an expression for the total area A_T of the triangular sections in terms of θ. **(2)**

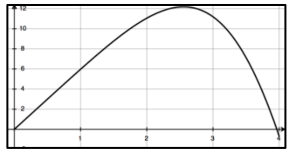
(c) Find how A_T is changing when $\theta = 60°$. Specify units. **(2)**

(d) Show that A_T is a maximum when $\theta = 45°$. **(1)**

(e) Write an expression for how fast the area A_R of the rectangular section is changing in terms of θ. Determine if A_R is at a maximum when $\theta = 90°$. Explain. **(3)**

7. (Calculator) Most cruise ships have passengers leave a sign outside their cabin door as to whether they want privacy or want their room to be serviced. Caribbean Princess has a panel outside the cabin door where the passenger can indicate that he or she wants the room serviced. Rooms are serviced starting at 8 AM until 12 PM. The number of rooms that have been serviced is modeled by $f(t) = 6t\cos\left(\dfrac{t^2}{10}\right), 0 \le t \le 4$ hours where $f(t)$ is measured in rooms per hour. The graph of f is shown below.





(a) If $F'(t) = f(t)$ and $F(0) = 0$, find $F(t)$. **(2)**

(b) Find the value and meaning of $F(3)$ in the context of the problem. **(2)**

(c) Explain why the cabin attendant is slowing down when $t = 3$. **(2)**

(d) Compare the average number of rooms the attendant clears in the first half of his shift as opposed to the 2^{nd} half. **(2)**

(e) Explain the seeming inconsistency between (d) and (e) in terms of the graph of *f*. **(1)**

8. (Calculator) On a large cruise ship like the Caribbean Princess, drinking water is supplied using a reverse osmosis process to desalinate sea water and treat it with chlorine and minerals. But for showering or using the toilet as well as doing laundry, ships carry massive tanks that can hold water for this purpose. This water is replenished at ports and while not dangerous, it is not recommended for drinking. Once used, it is treated before being released into the ocean. This is referred to as grey water. For this problem, dealing with water ultimately to become grey water, we will refer to it as grey water.

An 80,000-gallon tank of grey water is filled to capacity. At time $t = 0$, corresponding to 5 AM, to meet the needs of the ship, water drains out of the tank at a rate of $R(t)$ with t measured in hours and R is measured in gallons per hour. R is defined by the piecewise-defined function

$$R(t) = \begin{cases} \dfrac{7521t}{t+20}, 0 \le t \le 18 \\ 3898e^{-0.005t}, 18 < t \le 24 \end{cases}$$

(a) Using calculations to the nearest gallon, show that $R(t)$ is continuous at 11 PM. **(2)**

(b) Find the average rate of change which is grey water is being used between 6 AM and 12 midnight. **(3)**

(c) Find $R'(19)$. Using correct units, explain the meaning of that value in context of this problem. **(2)**

(d) The Caribbean Princess has 4 such tanks that get the same amount of water usage. How much grey water is left on the ship after 3 days? **(2)**

9. (Calculator) When people embark on a cruise ship such as the Caribbean Princess, they frequently have issues or questions as to their staterooms or accounts. Typically, they go to the Purser's desk which staffs 4 or 5 agents who can address their issues. On that first day, the desk is liable to be quite busy with people lining up to get service. The number of people in line at time t is modeled by a twice-differentiable function L for $0 \le t \le 5$ where t is measured in hours after 12 PM. Values of $L(t)$ are shown in the table below.

t (hours)	0	0.5	1	2	2.5	3	4	5
$L(t)$ people	2	15	25	18	28	15	5	1

(a) Use the data in the table to estimate the rate at which the number pf people waiting in line was changing at 2:45 PM. Show the computations that lead to your answer and indicate units of measure. **(2)**

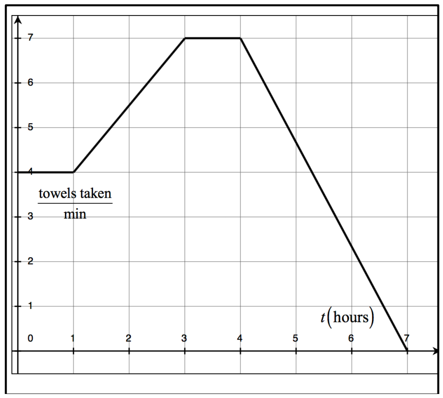
(b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line from 12 PM to 2 PM. **(2)**

(c) For $0 \le t \le 5$, determine whether it is possible that $L'(t)$ equals zero at 4 different values of t. Justify your answer. **(3)**

(d) For $0 \le t \le 5$, the rate that people had their issues resolved is modeled by $R(t) = 58t^2e^{-t/3}$ issues per hour. If the purser staff solved 62 issues for people prior to 12 PM, based on this model, how many total issues were solved by the time that the ship set sail at 5 PM? **(2)**

10. On a Caribbean cruise, guests typically spend a great deal of time on the pool deck which means the necessity of having many towels available. 500 folded towels are placed in the bins across the pool deck by 10 AM (time $t = 0$) in the morning. The graph below shows the rate $T(t)$ that people take towels from 10 AM through 5 PM measured in towels per minute and t is measured in hours.

Towels are washed and starting at 11 AM, towels are folded at the rate of 6 towels per minute and returned to the bins.



(a) How many towels are used during the 10 AM through 5 PM time period? **(2)**

(b) How many towels have been washed and folded through 5 PM? **(2)**

(c) At what time are all the towels washed and folded? **(2)**

(d) A deck officer makes a check every hour on the hour as to whether there are enough towels. Based on his checks, how many towels are available for use when this number is at a minimum? Justify your answer. **(3)**

11. (Calculator) Caribbean Princess has terrific Internet. One of the reasons is the installation of Mid-Earth Orbit (MEO) satellites. The principle is that the lower the orbit, the less distance from the ship to the closest satellite, reducing transmission time. These satellites are in orbit near the equator as there is no obstruction from natural objects like mountains. Further to the north, especially in the Alaska area, ships rely on GEO (geo-synchronous) satellites which are much higher.

In order to realistically create a problem using these concepts, we need to work in 3 dimensions as well as taking into account that ships follow the curvature of the earth. So let's simplify it. Assume that ships travel along a straight line and in the same plane as the satellites. We will change the actual height of the satellites by a factor of 10.

(a) The Caribbean Princess sails directly beneath a MEO satellite 480 miles high at 12:00 noon. How fast is the distance s between the satellite and the ship changing in mph at 4:00 PM and at 12:00 midnight if the ship sails at 18 mph? **(3)**

(b) The distance between the ship and the satellite is obviously increasing the further from the satellite the ship gets. Suppose there are 2,000 miles (straight line) between satellites. How many hours pass before the ship is furthest from a satellite and how fast is this distance changing at that time? **(2)**

(c) How fast is the angle of elevation (measured in degrees per hour) from the ship to the satellite changing at 12:00 midnight? **(4)**

12. Norovirus is an extremely contagious disease that can break out on a ship. One reason is that in the buffet, people are prone to touch the same surfaces – serving utensils, salt-shakers, and the like. If there is a suspected outbreak, ship's crews take measures like preventing people to self-serve at the buffet. Only when people self-report their symptoms can the ship's crew take measures to prevent further spread. Suppose Caribbean Princess is doing a two-week (14 day) Transatlantic cruise starting on Saturday when people start to get sick.

Day	Sick
Mon	20
Tues	28

The number of passengers with norovirus after 2 cruising days is given in the table. People don't self-report so no preventative measures are taken. The differential equation describing N, the number of people with norovirus is modeled by $\frac{dN}{dt} = kN$, with t measured in days.

(a) According to this model, how many people will have the virus by the end of the cruise? **(3)**

Now assume people self-report. After Tuesday, the staff takes preventative measures and the number of people with norovirus is now modeled by the DEQ $\frac{dN}{dt} = \frac{3N}{4}\left(1 - \frac{N}{100}\right)$ with the number of people who are actually sick given in the table to the right. Parts (b), (c) and (d) use this model.

Day	Wed	Thurs	Fri	Sat
Sick	45	62	75	90

(b) Use Euler's method with the given differential equation to find the discrepancy in the model (rounding to nearest person) and the real number of sick people for Monday through Saturday of the first week (6 days) . Consider Monday to be day 0. **(3)**

(c) Determine the minimum number of times there is no discrepancy between the model and the real number of sick people. Explain. **(1)**

(d) Using the model and the data, predict the day when half of the expected number of infected people for the entire cruise will have norovirus. Explain how you got your answer. **(2)**