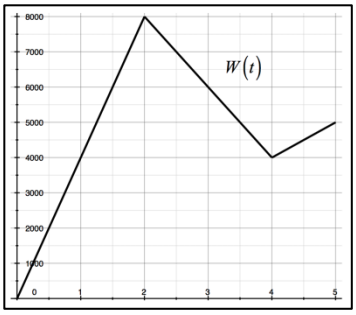
13. On the Caribbean Princess, many utensils are used in the dining rooms. At 5:00 PM, the start of the 4-hour time period when the restaurants are open for dinner, 5,000 utensils per hour are used in their sit-down restaurants. The rate that these dishes are washed and returned to the drawers is given by $W(t)$ as shown in the figure below where t is measured in hours. Tables are set and there are still 3,000 utensils in the drawers at the start of the dinner period.



(a) How many utensils were washed and returned to the drawers between 6:00 PM and 7:00 PM? Show the computations that lead to your answer. **(1)**

(b) Is the number of utensils on the tables increasing or decreasing between 6:00 and 7:00 PM? Justify your answer. **(2)**

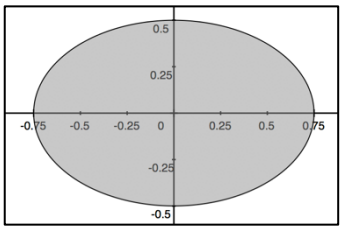
(c) At what time is the number of utensils available for use the smallest? Justify your answer. **(3)**

(d) How many utensils are available at that time? **(2)**

(e) How many utensils are in the drawers one hour after the dining room closes? **(1)**

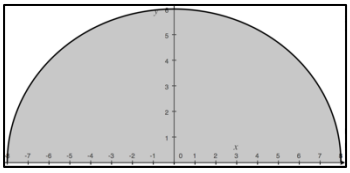
14. Area and volume don't usually lend themselves to cruising, at least not in passenger end of it. But the ship's chefs delight in not only providing delicious food, but also presenting it. So strange shapes come out of the kitchen. Here are two applications from the dining room.

(a) A piece of sushi is sliced in half lengthwise. Its footprint on the plate is in the shape of an ellipse as shown in the figure to the right. Its equation is $16x^2 + 36y^2 = 9$, where x and y are measured in inches. The area of the base can easily be found by measuring the ellipse's length a and its width b, both in inches, with the area being $\dfrac{\pi ab}{4}$. Write but do not calculate an integral that also determines this area.



(b) The 3-dimensional piece of sushi is created by rotating the base about the y-axis. Write but do not evaluate an expression that describes its volume.

(c) On the last night of a Princess cruise, it is traditional that Baked Alaska is served for dessert. This is an ice cream cake with sweet meringue on top that is placed into the oven for a short period to brown the meringue. A Baked Alaska has a base bordered by $\dfrac{x^2}{64} + \dfrac{y^2}{36} = 1, y > 0$ and the x-axis where x and y are measured in inches as shown in the figure to the right. The cake is constructed such that each slice is in the shape of a rectangle that is twice as high as it is wide. Write an integral expression representing the volume of the cake.



15. BC (Calculator) The Caribbean Princess dinner menu typically has about 6 – 8 main courses. How does ship staff know how much of each to have prepared? Using statistics from past cruises, ships can predict relatively accurately how many people will order a particular main course by creating a probability density function $P(x)$. The probability that between a and b people will order a particular main course is the first-quadrant area under this function from $x = a$ to $x = b$.

(a) Lobster is traditionally on the menu the last night of a Princess cruise. The probability density function for people ordering lobster on the last night of the cruise is given by $P(x) = 0.0015e^{-0.0015x}$ for $0 < x < \infty$. Determine whether P is increasing or decreasing and concave up or down as well as finding $\lim_{x\to\infty} P(x)$. Explain your answers. **(3)**

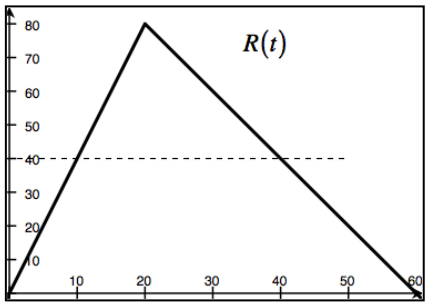
(b) What is the probability that between 200 people and 300 people will order lobster? Show how you get your answer. **(2)**

(c) The probability density function for people ordering lobster given in (a) is true for any Princess ship of any size. Write and evaluate an expression that determines the probability that 200 people or more will order lobster no matter how many passengers the ship holds. Show how you get your answer. **(2)**

(d) Every main course has a probability density function $P(x)$. If k is the number of passengers ordering that dish, write but do not solve an equation in terms of k such that the probability that k or more passengers order that dish is 50%. Then determine how fast this probability is changing at $x = k$. **(2)**

16. 

On the Caribbean Princess, the World Fresh Marketplace is a buffet with two main serving areas on each side of the ship. They serve breakfast from 7 AM through 11 AM. At 10 AM, the starboard side buffet closes to prepare for lunch while the portside stays open for another hour for late breakfast-comers. Let's concentrate on the portside buffet that final hour. For the first 50 minutes, bacon is cooked at the rate of 40 slices per minute and placed in warmers on the buffet line as shown by the dashed line in the figure to the right. With t measured in minutes, let $R(t)$ represent the rate, measured in slices per minute, that passengers take bacon from the warmers as shown by the solid lines in the graph. Suppose at the start of the 60-minute period, there were 450 slices of bacon in the warmers.



(a) Explain what is happening at 11:20 AM in terms of bacon consumption? **(1)**

(b) How many bacon slices are in the warmers at the end of breakfast-hour? **(3)**

(c) Write an expression for the number of slices of bacon in the warmers at time t, $0 \le t \le 5$. **(1)**

(d) At what time is there a maximum number of slices of bacon in the warmers? Show how you arrive at your answer. **(4)**

17. (Calculator) On Caribbean cruises, lots of time is spent in the warm sun and people flock to the pool decks where there are a great number of deck loungers. People known as "chair hogs" tend to come up to the pool deck early and stake a claim to a lounger in prime area by placing their bags and a towel on the lounger. They then disappear, sometimes for hours, going into the pool or for food. Someone coming up to the deck in the middle of the day is treated to the site of few people sitting in the loungers but most of them claimed. Cruise management posts signs discouraging this behavior but it is usually ignored.

The Caribbean Princess has 480 loungers on its pool decks. One sunny day, the number of chairs claimed with bags on them is modeled by a differentiable function $C(t)$ for $0 \le t \le 7$ where $t = 0$ corresponds to 10 AM. Values of $C(t)$, in hundreds of chairs at various time t are shown in the table below.

t (hours)	0	2	4	6	7
$C(t)$ hundreds of chairs	0	2.45	4.45	5.6	4.8

(a) Use the data in the table to approximate the rate in chairs per hour at which chairs are claimed at 3:00 PM. Show the computation that leads to your answer. **(1)**

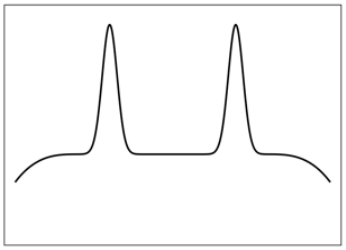
(b) Use a trapezoidal sum with four subintervals given by the table to approximate the value of $\frac{1}{7}\int_0^7 C(t)\,dt$. Using correct units, explain the meaning of this expression in terms of the number of chairs claimed. **(2)**

(c) At 5:00 PM, there is a report from the bridge of strong winds which will blow the loungers around creating a dangerous situation. They need to be stacked and tied down for the night. An army of deckhands begin to move and stack the chairs. They do so at a rate of $S(t) = 0.5t^3 - 12t^2 + 95.5t + 210$ for $t \ge 7$. According to the model, how many chairs had not been stacked by 6:00 PM? **(3)**

(d) According from the model from part *c*, at approximately what time were the chairs being stacked most quickly? Explain how you got your answer. **(3)**

18. 

(Calculator) People like to work out in the extensive Fitness Center on the Caribbean Princess. On a treadmill, Ken chooses the workout shown to the right. This indicates that he will have a warmup period and then be forced to running faster, then slowing down before running at a constant pace. This more intense section is repeated before the runner warms down.



If a 30-minute workout is chosen, the first half of the workout W is given as a function of t, where t is measured in minutes and $W(t)$ is measured in miles per hour.

$$W(t)=\begin{cases}5+0.01(t-6)^3, t\le 6\\ 5+10e^{-(t-9)^2}, 6<t<12\\ 5, 12\le t\le 15\end{cases}$$

(a) How much further does Ken run in the more intensive sections than the other periods? Explain how you get the units. **(3)**

(b) At what time(s) in the workout is Ken's acceleration the greatest? Show how you got your answer. **(3)**

(c) Ken's pulse rate is above 125 beats-per-minute when he is running at 12 mph or faster. What percentage of Ken's total mileage is run with a pulse rate of at least 125 beats per minute? Is this the same as Ken's percentage of time run with a pulse rate of at least 125 beats per minute? **(3)**

19. (Calculator) The Caribbean Princess has a pool on the very back of the ship called the Terrace Pool. We will say that it is the shape of a square with a semi-circle of radius 16 feet (see picture). One evening it was emptied of water as the crew was repainting the blue Princess logo at the bottom as well as the sides of the pool. The paint at the bottom dried quickly and 3 inches of water was put in the pool to protect it overnight. When the entire pool was completely filled at 5 AM, the height of water in the pool was modeled by the function h, measured in feet and t is measured in hours after 5 AM for $0 \le t \le 3$. The rate of at which the height of the water in the pool is rising is given by $h'(t) = 2.85 - 12\sin(0.08t)$.

(a) According to the model, what is the height of the water in the pool after 3 hours? **(3)**

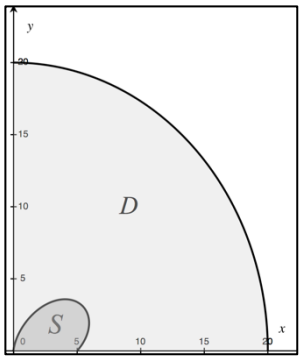
(b) According to the model, what is the average change in height of the water in the pool over the 3 hours time period? Show the computations that lead to your answer and indicate units. **(1)**

(c) At what rate is the volume of the water in the pool changing at 7:00 AM? Indicate units of measure. **(3)**

(d) The Sanctuary pool towards the bow of the ship is also being filled in the same 3-hour period. The height of the water is modeled by the function s where $s(t) = \frac{1}{10}(2t^3 - 3t^2 + 3t + 5)$, where s is measured in feet. Use the Intermediate Value Theorem to show that there exists a time k in that 3-hour period at which the heights of the water in the 2 pools are changing at the same rates. **(2)**

20. 

BC (Calculator) The Caribbean Princess is having a deck party at night. The graph shows the polar curve $f(\theta)=5(1+\sin\theta\cos 2\theta)$ and $g(\theta)=20,\ 0\le\theta\le\pi/2$.



Let S be the stage area (where the DJ works) in the first quadrant bordered by $f(\theta)$ and the x-axis. Let D be the dance area in the first quadrant bordered by $f(\theta)$, $g(\theta)$, and the x- and y-axes. f and g are measured in feet.

(a) Find the dancing area. **(2)**

(b) It is decided to put a narrow aisle along angle $\theta=a$ where $0<a<\pi/2$ that will divide the dancing area into two regions of equal area. Write, but do not solve and equation involving one or more integrals whose solution gives the value of a. **(2)**

(c) For each value of $\theta,\ 0\le\theta\le\pi/2$, let s be the distance between the very back of the dancing area and the very front of the stage. Write and expression for $s(\theta)$ and find s_{avg}, the average value of $s(\theta)$. **(3)**

(d) Using the answer from part (c) find the angle θ for which $s(\theta)=s_{avg}$. Determine whether $s(\theta)$ is increasing or decreasing at that angle. Justify your answer. **(2)**

21. (BC) On all cruise ships like Caribbean Princess, there is a casino and it attracts a great number of people. It is only open for business when the ship is at sea. The odds are very much against passengers at a ship's casino (because there is no competition) but that doesn't discourage people from gambling and the casino typically makes a great deal of money.

A cruise leaves port at 5 PM and its casino opens at 6 PM. Define a function R that represents the rate of profit at the casino measured in dollars per hour as a function of t, where t represents hours after 8 PM.

$$R(t) = 6000\left(\frac{1}{2} + \frac{2t}{2^2} + \frac{3t^2}{2^3} + \ldots + \frac{n+1}{2^{n+1}}t^n + \ldots\right)$$

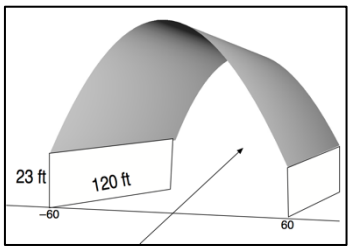
(a) Find the interval of convergence of the given power series and explain its meaning in the context of the problem situation. **(4)**

(b) Find $\lim_{t\to 0}\dfrac{R(t)-3000}{t}$ **(1)**

(c) Write the infinite series that represents $\int_0^1 R(t)\,dt$, showing the first 3 terms. **(2)**

(d) Find the sum of the series determined in (c) and explain its meaning in the context of the problem. **(2)**

22. (BC) (Calculator) Many Princess cruise ships have their Neptune pool covered. Caribbean Princess, which was named because it was meant to pretty much stay in the Caribbean does not because of the mostly good weather that it sees. However, when bad weather occurs, passengers are upset as they cannot swim or watch movies and a large area is pretty much unavailable. Let's suppose that Princess management decides to enclose the area with a small glass dome.



A dome that is 120 feet long and 120 feet wide is built over the Neptune pool. A cross section of the dome is the curve $y = 37 - 12\left(e^{x/60} + e^{-x/60}\right)$ as shown by the figure on the left. It is built over the side walls of the deck which are each 120 ft long and 23 feet high.

(a) If the edges to the dome including where it intersects the walls of the deck are to be painted in a wide stripe, what is the total length of the stripe in feet? Show how you get your answer. **(3)**

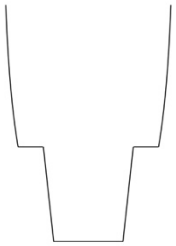
(b) If the area of the dome is to be covered in glass panels, what square footage is required? **(1)**

(c) Find the total airspace under the dome. **(3)**

(d) A rectangular grid for lights is to be constructed 5 feet from the top of the dome. What is the area of the grid? **(2)**

23. One of the more popular areas on Caribbean Princess is the 7 hot tubs. The hot tubs are a fun way to get wet and relax and are kept at a temperature where people can stay in comfortably for a while. Unfortunately, bacteria thrive in hot tubs and ship's staff completely empty hot tubs every night and refill them in the morning.

A hot tub has a height of 5 feet. It has a lower section, and an upper section as pictured to the right. The seat is at height 2 feet. The area of the horizontal cross section of the tank at height h feet in each section is given by the function A where $A(h)$ is measured in square feet. In each section, A is continuous and increases as h increases. Selected values for $A(h)$ are given in the table below with the values of A at $h = 2$, the area of the cross section just below the seat and just above it.



h(feet)	0	1	2	2	3	4	5
$A(h)(\text{ft}^2)$	20.3	22.7	26.4	40.8	44.4	46.6	48.0

(a) Use a left Riemann sum for both the lower and upper section to approximate the volume of the hot tub. Indicate units of measure. **(1)**

(b) Does the approximation in (a) overestimate or underestimate the volume of the hot tub? Explain your reasoning. **(1)**

(c) The area, in square feet, of the horizontal cross section at height h is modeled by the function f (lower) and g (upper) as given by $f(h) = 6.1h + 13.82$ and $g(h) = 50 - \frac{25}{\sqrt{e^x}}$. If there is 1 gallon in 0.1337 ft^3, and the tub is filled to 3 inches below the top, how many gallons of water does the hot tub hold? **(3)**

(d) The hot tub is filled at the rate of 40 gallons per minute. Find how fast the water in the tub is rising when there is 1 foot of water in the tub. Specify units of measure. **(4)**