24. BC (Calculator) Many people go on cruises to drink. But it can be expensive. Princess Beverage packages give pretty much unlimited drinks daily but if one person in a stateroom gets the beverage package, everyone must also purchase it to eliminate sharing of drinks. If you purchase a beverage package when you book the cruise, you can frequently get the best possible deal. At a point in time before the cruise, the daily price rises every day.

Suppose you (solo traveler) had a choice of purchasing a beverage package at \$55 a day when you booked the cruise. That price was valid until 30 days before the cruise and then the daily price rose at the rate of

$$f(t) = 0.02t + 0.6, -30 \le t \le 0. \text{ Let } F'(t) = f(t) \text{ with } F(-30) = 55.$$

(a) Write an expression for $F(t)$. **(1)**

(b) Use the results in (a) to find $F(0)$ and explain your result in the contest of the problem situation. If you integrate, show the result of your integration before you use the calculator. **(3)**

You have a choice of choosing another beverage package option when you book the cruise. You still pay \$55 a day and that price is valid until 60 days before the cruise and then the daily price rose at the rate of

$$g(t) = 0.01\left[\sqrt{t+61} \cdot \ln(t+61)\right] - 60 \le t \le 0. \text{ Let } G'(t) = g(t) \text{ with } G(-60) = 55.$$

(c) Write an expression for $G(t)$. **(1)**

(d) Use the results in (c) to find $G(0)$ and explain your result in the contest of the problem situation. If you integrate, show the result of your integration before you use the calculator. **(4)**

25. (Calculator) Princess ships uses medallions to identify passengers. Wearing a medallion allows you as a passenger to use an app to order drinks. You order and a staff member delivers the drink you as they can track your exact location when you wear your medallion. Still, the process can take time. The amount of time to get your drink is a function $T(x)$ where x is your place in the line among people who did not receive their drinks. For one location on the ship close to a bar, $T(x) = 0.8\sqrt{x}$ minutes per person. Some people complain about the time and would rather just wait in line for service at a bar where it takes 2 minutes on average to serve a drink.

(a) Find the value of $\int_5^{10} T(x)\,dx$ and explain its meaning in the context of this problem. **(2)**

(b) Which is the shorter wait (medallion or bar) when you are 4^{th} in line? Show how you got your answer. **(1)**

(c) Write an expression involving an integral that represents the savings in time when you are the *k*th position in line, whether you wait or use the medallion. **(1)**

(d) Approximately how many people must be there be in line for the wait time for the two methods of ordering drinks to be the same? Show your reasoning. **(3)**

(e) Find the value of x for which $T(x) = 2$. Explain the significance of this value and values around it in the context of the problem. **(2)**

26. (Calculator) On most cruise ships, there is a pizzeria on the pool deck for passengers who don't want to go into the buffet and just want a quick snack. The two attendants are constantly assembling pizzas, taking them in and out of ovens, slicing them, and serving to passengers in any quantity they desire. It takes a lot of cheese and sauce to accomplish this and the pizzeria must be constantly replenished.

The Slice Pizzeria on Caribbean Princess opens at 10 AM, and although it stays open for 14 hours, it does most of its business between 10 AM and 4 PM. Cheese for the pizza is taken from 4 large 50-pound containers. For $0 \le t \le 5$, the amount of cheese remaining in the containers is modeled by $C(t) = 200 - 0.4\ln(t+1)e^t$, where $C(t)$ is measured in pounds and t is measured in hours past 10 AM.

(a) Find the average rate of change of $C(t)$ from 10 AM to 3 PM. Indicate units of measure. **(1)**

(b) Find $C'(t)$ and use it to find $C'(2)$. Using correct units, interpret the meaning of the value in the context of the problem. **(3)**

(c) Find the time of day when the amount of cheese in the containers is equal to the average amount of cheese in the container from 10 AM to 3 PM. **(2)**

(d) After 3 PM, $L(t)$, the linear approximation to C at 3 PM is a better model for the amount of cheese remaining in the containers. Use $L(t)$ to predict the time when there is no remaining cheese in the containers and more will have to be brought to Slice. **(3)**

27. (Calculator) On the Caribbean Princess, there is always a lineup for soft serve ice cream and it will quickly melt in the hot sun of the Lido deck. At time $t = 0$, a cone is given to a male passenger who takes 2 minutes to deliver it to his wife. The temperature of the ice cream coming from the machine is initially $16°$. The temperature of the ice cream while it is still in a frozen state is modeled by the function T which satisfies the differential equation $\frac{dT}{dt} = \frac{1}{8}(32 - T)$ where T is measured in degrees Fahrenheit and t is measured in minutes.

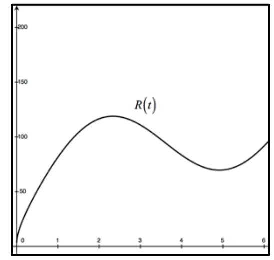
(a) Write an equation for the line tangent to the graph of T at $t = 0$. Use this equation to approximate the temperature of the ice cream at $t = 2$. **(2)**

(b) Use $\frac{d^2T}{dt^2}$ to determine whether your answer in part (a) is an overestimate or underestimate of the actual temperature of the ice cream at time $t = 2$. **(1)**

(c) While the cone is not fully melted, determine the equation that describes the temperature of the ice cream at time t and use it to find its temperature when his wife receives it. **(4)**

(d) The ice cream will drip uncontrollably when it reaches $24°$. Determine how long his wife has to finish the ice cream. **(2)**

28. 



BC (Calculator) On the Caribbean Princess, the International Café, open 24 hours a day, is available for people who just want light fare instead of a full meal. From 5 AM through 11 AM, it has breakfast items available and people choose to go there rather than the crowded buffet. In order to stock it properly, corporate staff approximate the rate R that people historically order breakfast items from the Café which is is modeled by $R(t)=40\left(0.5+\sqrt{t}+\cos(t-2)\right)$ where t is measured in hours after 5 AM and R is measured in people per hour as shown by the graph above. Because ship staff only are equipped with 4-function calculators, they decide to approximate R using polynomials.

(a) Find a 2nd degree Taylor polynomial centered at $t = 1$ to approximate $f(t)=\sqrt{t}$. **(3)**

(b) Find a 4th degree Taylor polynomial centered at $t = 2$ to approximate $g(t)=\cos(t-2)$. **(1)**

(c) Combine (a) and (b) to find the approximation $A(t)$ to $R(t)$. **(1)**

(d) Use your calculators to determine the times of the day when the model and the approximation to the model give the same rate of change in the number of people ordering at the International Café. **(2)**

(e) Determine the number of people the model and the approximation to the model predict will show up at the International Café between 5 AM and 11 AM. **(2)**

29. On Caribbean Princess, the place for a burger and fries is the Salty Dog Grill. Passengers line up and place their order. Because the number of people waiting can get long at lunchtime, the chefs have burgers partially cooked in a drawer under the grill and place them on the grill to further cook when they are ordered. For safety reasons, they are cooked until they are done medium-well. For cheeseburgers, cheese is placed on the top of the burger when served.

When a passenger orders them, hamburgers at a temperature of $100°\text{F}$ are placed on a grill which will change the internal temperature of the meat according to the differential equation $\frac{dT}{dt} = \frac{250-T}{10}$ where T is measured in degrees Fahrenheit and t is measured in minutes.

(a) Find the slope of the line tangent to the graph of T at $t = 0$. Use this to approximate the raise in internal temperature of the burger for every minute that it stays on the grill. **(1)**

(b) Solve the differential equation using the initial condition of $T(0) = 100$. **(4)**

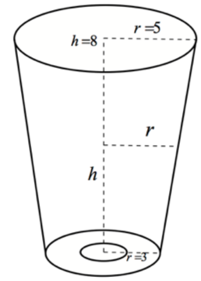
(c) The Grill's chefs have instructions to serve the burger when it reaches $150°\text{F}$. How long will the burger cook? Express your answer mathematically and then accurate to the nearest minute/seconds. **(2)**

(d) I request that the cheese be melted on the burger while cooking. The chefs put the cheese on after it reaches the safe temperature of $150°\text{F}$ and the cheese takes an extra 90 seconds to melt while the burger continues to cook. What is the internal temperature of the burger when I receive it? **(1)**

(e) Find $\lim_{t\to\infty} \frac{dT}{dt}$ and its meaning in the context of the problem. **(1)**

30. 

Drinking is big business on a cruise ship and there are bars galore. All of them have multiple blenders. One of them on the Caribbean Princess has a hole in the bottom so that when a thick drink is mixed, the bartender sets takes the jar from the housing and places it on top of a glass so the drink will drain slowly and he can do something else. The bottom radius of the blender is 3 inches and the top radius at height $h = 8$ inches is 5 inches. The sides are straight.



(a) Write an expression that represents the radius as a function of the height of the blender. **(1)**

(b) Show that the average value of the radius of the blender is 4 inches. **(2)**

(c) Find the volume of the blender. Show how you get your answer. **(3)**

(d) Write but do not solve an equation that determines the height of the drink s when the blender is half-full. **(1)**

(e) The blender contains a thick pina-colada drink. When the height of the drink is $h = 4$ inches, the radius of the surface of the drink is decreasing at the rate of 1/6 inch per second. At this instant, what is the rate of change of the height of the drink with respect to time? **(2)**

31. BC (Calculator) Caribbean Princess visits Princess Cays, a private island where people can have a beach experience as well as do water sports in the Caribbean and eat and drink. There is no port so the ship anchors offshore and people who choose to visit the island are tendered off the ship. Tenders leave the ship periodically between 8 AM and 1 PM and tenders return to the ship between 10 AM and 3 PM. (Note: since people leave the ship and return by tenders, they travel in groups the size of tender capacity. So the functions described below are not continuous. For the sake of the problem topic, we will assume that the functions are indeed continuous and differentiable).

(a) People leaving the ship is modeled by a function L that satisfies the separable differentiable equation

$\frac{dL}{dt} = \frac{L}{2}\left(1 - \frac{t}{5}\right)$ where L is measured in hundreds of people and t is the number of hours after 8 AM. If 380 staff are initially tendered off the ship to the island to set up, find $L(t)$. **(5)**

(b) How many total people have left the ship after the last shuttle to the island leaves? **(1)**

(c) The number of people returning to the ship is modeled by a function R that satisfies the differential equation $\frac{dR}{dt} = \frac{3R}{2}\left(1 - \frac{3R}{40}\right)$ where R is measured in hundreds of people and t is the number of hours after 10 AM. If 100 people have returned by the time this shuttle begins regular service, what is $\lim_{t\to\infty} R(t)$? Explain your answer. **(2)**

(d) How many people have returned when the number of people returning is increasing the fastest? **(1)**

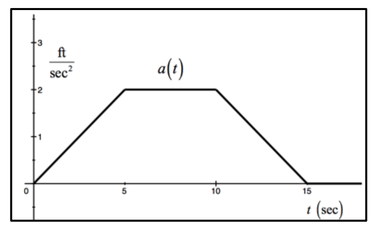
32. The Caribbean Princess visits the island of St. Maarten. The ship docks at the straight-line pier and people get off and walk its length (about a half-mile) until they reach the security gate. Because many cruisers are older, there are also trams that run along the pier to transport people to the gate or return to the ship. Typically though, these trams are not operating for people with early morning excursions, so they are forced to walk. There are places to sit along the pier.

(a) I get off the ship early and must walk the pier to the security gate. My velocity v which is a differentiable function measured in feet/sec is given every minute as shown by the table to the right. Using the trapezoidal rule using all of these values, what is my average speed? Express using proper units. **(2)**

$t(\text{min})$	0	1	2	3	4	5	6	7	8
$v(\text{ft/sec})$	0	5	6	4	1	7	5	3	2

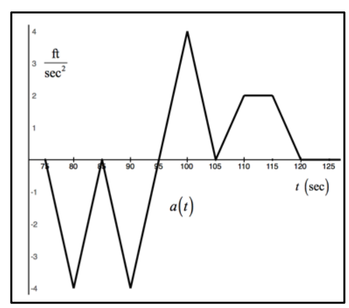
(b) Between what two times does Rolle's Theorem state that I am not accelerating? **(1)**

(c) On the pier going back to the ship, I take a tram that is parked just inside security. Its acceleration, measured in $\frac{\text{ft}}{\text{sec}^2}$ is shown in the figure to the right. How fast is the tram traveling after 15 seconds? **(2)**



(d) The tram continues for one-minute with acceleration 0 when the driver realizes that a bag fell off the tram and he needs to back up to retrieve it. The acceleration of the tram for the next part of the ride is shown in the figure to the right. Explain the motion and speed of the tram in the following time durations, measured in seconds. **(4)**

i) (75,85) ii) (85,95) iii (95,105) iv) (105, 120)



33. (Calculator) Passengers embarking and debarking at major ports with 2-story terminals typically have glass-enclosed gangways that are barely sloped making it easier to walk on and off the ship. But at smaller ports, the gangways down to the dock can be surprisingly short and angled making walking difficult. Since a ship like Caribbean Princess is typically in port for over half a day, it experiences the changing tides, lowering and raising the ship, which affects the length and angle of the gangway.

(a) People debark at St. Thomas when it is low tide at 8 AM. The debarkation gangway, 25 feet in length, leads to a ship door just 7 feet about the level of the dock. What is the angle of elevation of the gangway in degrees? **(1)**

(b) If the tide is coming in and the ship is being raised by 1.5 ft/hr, how fast is the angle of elevation of the gangway changing at 10 AM in degrees per hour? **(4)**

(c) The maximum gangway angle that passengers are allowed to walk is $30°$. At what time will that occur Assuming the ship is still raised at 1.5 ft/hr. **(1)**

(d) The gangway telescopes so it can be made longer. When the angle of elevation of the gangway reaches $30°$, it will be continuously extended to maintain the angle of $30°$. How long is it and how fast is it being extended at high tide, 4 PM? **(3)**