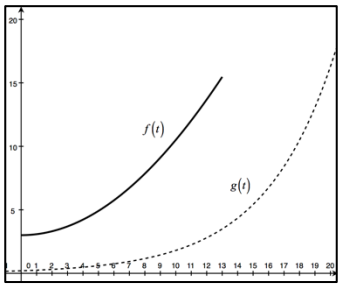
34. (Calculator) The Explorer's Lounge on the Caribbean Princess is a smaller venue for cruise game shows, lectures, or concerts. It is semi-circular with a small stage. While always open, it typically fills up starting 20 minutes before a scheduled event. A popular show begins at 8:30 PM and most people arrive starting at 8:10 PM. Passengers enter through the back near the bar and seats tend to fill up from the back to the front as people don't like threading their way through tables.

The rear section, with 5 people sitting there initially takes 13 minutes to completely fill up and does so at a rate modeled by

$$f(t) = 3 + 5t\sin\left(\frac{t}{52}\right), 0 \le t \le 13$$



where t is measured in minutes after 8:10 PM and $f(t)$ is measured in people per minute. The front section, with 4 people sitting there initially fills up at a rate modeled by

$$g(t) = 0.2e^{0.22t}, 0 \le t \le 20$$

where t is measured in minutes after 8:10 PM and $g(t)$ is measured in people per minute.

(a) Write a piecewise expression $F(t)$ representing the number of people sitting in the rear section at time $t = k$, $0 \le k \le 20$. Use it to find and interpret $F(11)$. **(3)**

(b) Using proper units, find and interpret $f'(11)$. **(2)**

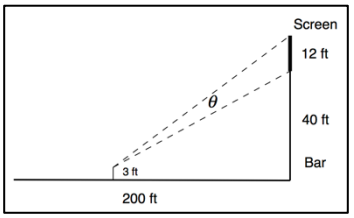
(c) Write an expression representing the difference between the number of people sitting in the rear section and front section at time t, $0 \le t \le 13$. Is this difference increasing or decreasing at $t = 11$? Give a reason for your answer. **(3)**

(d) If the lounge is completely full when the show begins, how many people does it hold? **(1)**

35. 

(Calculator) On the Caribbean Princess, there is a large screen that shows movies on the outside deck. While movies and videos are on most of the day, first run films are shown in the evening and it is called "Movies Under the Stars." Deck loungers are set up with blankets and popcorn is available.

Suppose a deck 200 feet long has a large screen that measures 25 feet long and 12 feet high. The bottom of the screen is 40 feet above the deck. Under the screen is a bar as shown in the figure to the right.



(a) If Mike walks away from the bar at 3 ft/sec, how fast is his distance from the middle of the screen changing 5 seconds later? Show how you got your answer. **(2)**

(b) Let θ be the viewing angle of the screen (bottom of screen to top) from wherever Mike sits, x feet from the bar. Assume he is 3 feet off the ground when sitting. Write an expression representing angle θ and use it to calculate θ in degrees when he is sitting 15 feet from the bar. **(2)**

(c) Using information from (b) determine how fast this angle is changing in degrees/sec if Mike walks at 3 ft/sec? Show how you get your answer. **(3)**

(d) The best seat is the one that maximizes the angle of the screen. How far should Mike sit away from the bar to have the best seat? **(2)**

36. (Calculator) Most cruise ships have a show in the theatre in the evening. There is usually an early show and a late show and people arrive early for a good seat, which surprising is in the back of the theatre, so people don't have to climb steps when leaving.

On Caribbean Princess, the late show begins at 9:30 PM and people are admitted to the theatre at 9:00 PM. The rate that people enter the theatre for a Broadway Revue show is modeled by the function $B(t)$ given by $B(t) = 0.012\left(33t^2 - t^3\right)$ for $0 \le t \le 30$ minutes. No one is in the theatre at 9:00 PM.

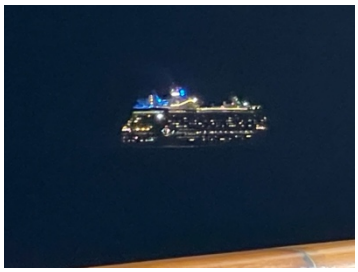
(a) How many people are in the theatre when the show begins? Show how you got your answer. **(2)**

(b) Find the time when the rate at which people entering the theatre is at a maximum. Justify your answer. **(2)**

(c) The total wait time for all the people in the theatre is found by adding the time each person waits, starting when the person enters the theatre and ending when the show begins. The function W models the total wait time for all the people who enter the theatre before time t. The derivative of W is given by $W'(t) = (30 - t)B(t)$. Find $W(30) - W(15)$, the total wait time for those who enter the theatre after 9:15 PM. **(2)**

(d) On average, how long does a person wait in the theatre for the show to begin? Consider all people who enter the theatre after the doors open and the model for total wait time from part (c). Show how the units evolve to minutes. **(3)**

37. (Calculator) At 7:00 PM, the Caribbean Princess sails along a straight line with position $x_C(t) = 42 - 40e^{-0.5t}$, t measured in hours from 7:00 PM. Its sister ship, the Star Princess, parallels her 4 miles away with velocity $V_S(t) = 0.3(3-2t)^2$. At $t = \frac{1}{2}$, $x_S(t) = \frac{3}{5}$.



(a) Find $V_C(t)$, the velocity of the Caribbean Princess at time t. **(1)**

(b) Find $a_S(t)$, the acceleration of the Star Princess at time t. For times $0 < t \leq 4$, find the times when the speed of the Star Princess is decreasing. Justify your answer. **(2)**

(c) Find $x_S(t)$, the position of the Star Princess at time t. **(2)**

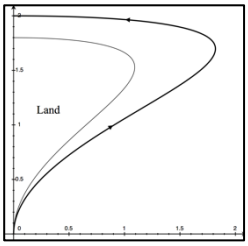
(d) Write an expression representing the horizontal distance between the two ships at time t. Use it to find this distance at 8:00 PM. **(2)**

(e) Determine how fast the ships are separating at that time. **(2)**

38. BC (Calculator) All cruise ships require a pilot who has an intimate knowledge of the nuances of the port waters to help guide the ship in and out of port

With the help of a local pilot, the Caribbean Princess navigates out of port at the island of St. Thomas and sails around a promontory (land jutting out into water) before going out to sea. For $0 \le t \le 2\pi$, t measured in minutes, the ship moves along the curve shown below so that its position at time t is $s(t) = \langle x(t), y(t) \rangle$ where $x(t) = 0.5t \sin(0.5t)$ and $y(t) = 2\sin(0.25t)$ and x and y are measured in kilometers. After the ship rounds the promontory, a pilot ship meets the ship, collects the pilot, and goes back to port.

(a) Determine the ship's position at $t = 4$ and locate it on the figure below. **(1)**



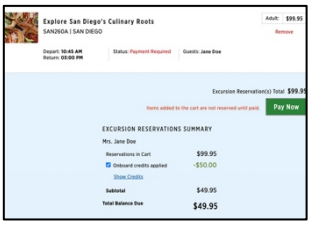
(b) Determine the velocity vector at $t = 4$ and locate it on the figure above. **(2)**

(c) Show that the ship is barely moving at $t = 4$. Explain why this is happening. **(2)**

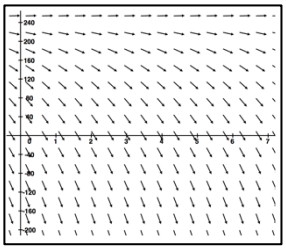
(d) The pilot ship collects the pilot at $t = 5$ and moves off tangent to the ship's motion. Determine the slope of its motion. **(2)**

(e) Find the total distance that the ship travels in 2π minutes. Show the setup for your calculation. **(2)**

39. When passengers book a cruise, there are deals available that give them incentives in terms of onboard credits that can be used on the ship. These credits can be spent in gift shops, specialty restaurants, excursions, the spa, and other locations. However, these credits cannot be transferred to future cruises. This results in people scrambling to use their credits by the end of the cruise. If passengers use more than their allotted credits, they must pay their balance by the end of the cruise.



The Caribbean Princess left for a 7-day cruise on Saturday. The average amount of funds available as credits for the passengers of this cruise is modeled by the differential equation $\frac{dF}{dt} = \frac{F}{2} - 125$, t neasured in days. When the cruise begins, the average passenger had \$240 in credits. We will assume for this cruise that no passenger gains credits once the cruise begins.



(a) For the slope field for $\frac{dF}{dt} = \frac{F}{2} - 125$ shown to the right, sketch the solution curve through the point (0, 240). **(1)**

(b) Use the line tangent to the graph of F at the start of the cruise to approximate the average amount of funds that a passenger has available on Monday. **(2)**

(c) Write an expression for $\frac{d^2F}{dt^2}$ in terms of F. Use $\frac{d^2F}{dt^2}$ to determine whether the answer in (b) is an underestimate or overestimate for the actual value of F. Give a reason for your answer. **(2)**

(d) Find the particular solution to the differential equation $\frac{dF}{dt} = \frac{F}{2} - 125$ with initial condition $F(0) = 240$. Use this result to predict whether the average passenger owes money at the end of the cruise. **(4)**

40. (Calculator) When the cruise is over and the ship arrives in its final port, people who live locally usually carry off their own luggage. They can debark once the ship is cleared by local officials. People like to line up to be among the first off the ship.

Caribbean Princess docks at 7:00 AM and it is estimated that passengers can leave the ship at 8:00 AM. Suppose people carrying their own luggage join the line to get off the ship at a rate modeled by the function R given below where t is measured in minutes after 7:00 AM.

$$R(t)=\begin{cases}204\left(\frac{t}{20}\right)^3\left(1-\frac{t}{60}\right)^6, 0\le t\le 45\\ 0, t>45\end{cases}$$

At 7:25 AM, Caribbean Princess is cleared early and people start to leave the ship. However there is only one scanning device on hand so they exit at the rate of only 10 people per minute.

(a) If 56 people were already in line at 7:00 AM, how many people have lined up to carry their own luggage off the ship in by 7:45 AM? **(2)**

(b) How many people are in line at 7:45 AM? **(2)**

(c) To the nearest minute, at what time will there be no one waiting in line to get off? **(1)**

(d) At what time (nearest minute) is the number of people in line a maximum? How many people are in line at that time? Justify your answer. **(4)**