

DIGITLE – AB CALCULUS

Puzzle 113 – Three Important Theorems



Directions: The first 5 problems have single digit answers. The 6th problem has a 5-digit answer (counting leading zeros if present). You have a choice: solve the easier single-digit answer problems or tackle the more difficult 5-digit answer. Once you have done that, attempt to solve the puzzle by entering the following url on your computer, tablet, or phone:

<https://mastermathmentor.com/mmm/digitle.ashx>.

The correct puzzle answer will be the digits of your answer(s) scrambled. Use the following interpretation. You get 6 tries.

Green : the digit is in the answer and is in the correct spot.
Yellow: the digit is in the answer but is not in the correct spot.

Grey : the digit is not in the answer.

Single Digit Answers:

- 1) For what value of x does Rolle's Theorem hold between the positive roots of $f(x) = 27x - x^3$?
- 2) The intermediate value theorem states that if the function $f(x)$ is defined on the interval $[a, b]$ and k is a number between $f(a)$ and $f(b)$, then there must be at least one value c within $[a, b]$ such that $f(c) = k$. If $f(x) = 2\cos\frac{x}{2}$, $a = 0$, $b = 10\pi$, and $k = -1$, at how many locations does the IVT hold (use zero if the conditions are not met).
- 3) (Calculator) How many values of c satisfies the Mean-Value Theorem for $f(x) = xe^{-x}$ on $[-1, 1]$?
- 4) The function f is differentiable $f\left(\frac{1}{8}\right) = \frac{1}{2}$ and $f\left(\frac{1}{2}\right) = 2$. For some k where $\frac{1}{8} \leq k \leq \frac{1}{2}$, $f'(k)$ must equal what number?
- 5) Function f is differentiable. Differentiable function g on $[-1, 2]$ is given by $g(x) = f(f(x))$. The table below gives selected values of f and f' . Find the minimum number of horizontal tangent lines to g on $[-1, 2]$?

x	-1	0	1	2
$f(x)$	1	-1	2	0
$f'(x)$	-3	4	-2	e

5-Digit Answer:

- 6) The graph of $y = e^{\cos\pi x}$ is shown on the figure on the right. The number of locations on the interval $[0, 2n - 1]$ where n is an integer is calculated where the mean-value theorem holds? If $n = 17562$, what is the result of this calculation?

