DIGITLE – BC CALCULUS
Puzzle 210 – Vector-Valued Functions

Directions: The first 5 problems have single digit or letter answers. The 6th problem has a 3-digit answer (counting leading zeros if present). You have a choice: solve the easier single-character answer problems or tackle the more difficult 3-digit answer and the multiple choice.

Single Digit or Letter Answers:

1) A particle moving along a curve in the *xy*-plane has position $(x(t),y(t))$ at time $t \ge 0$ as shown below. For what value of t is the particle stopped?

$$\frac{dx}{dt} = e^{t^2-8t+15} - 1 \quad \text{and} \quad \frac{dy}{dt} = \sin\left(\frac{\pi t}{2}\right)\cos \pi t + 1$$

2) If $\mathbf{u}(t) = \left\langle e^t, \frac{1}{\sqrt{t}} \right\rangle$ and $\mathbf{v}(t) = \left\langle \frac{1}{t}, t \right\rangle$, find $D_t\left[\mathbf{u}(t)\cdot\mathbf{v}(t)\right]$.

A. $\left\langle \frac{te^t - e^t}{t^2}, \frac{1}{2\sqrt{t}} \right\rangle$ B. $\left\langle e^t, \frac{1}{2\sqrt{t}} \right\rangle$ C. $e^t + \frac{1}{2\sqrt{t}}$ D. $\frac{te^t - e^t}{t^2} + \frac{1}{2\sqrt{t}}$

3) A particle moving along a curve in the *xy*-plane has position $(x(t),y(t))$ at time $t > 0$. The velocity of the particle is $\mathbf{v}(t) = \left\langle 2 - \frac{3}{t^2}, 1 + \frac{1}{t^2} \right\rangle$. For what time $t \ge 0$ does the line tangent to the path of the particle at $(x(t),y(t))$ have a slope of -2?

4) (Calculator) Two particle are moving along a curve in the *xy*-plane. The first has position $(x(t),y(t))$ at time $t \ge 0$ with $\frac{dx}{dt} = 2\cos\left(e^{t/2}\right)$ and $\frac{dy}{dt} = \ln\left(t^2 - 2t + 4\right)$ with $\left(x(0),y(0)\right) = (0,4)$. The second has speed $0.4t + 1.2$ with $\left(x(0),y(0)\right) = (4,0)$. On the interval $0 \le t \le 5$, how many times are the speeds of the particles the same?

5) A particle is moving along a curve such that its velocity at time t is $\left\langle e^{-t}, \frac{8}{t^2+1} \right\rangle$. If the particle is at the position $(0,-2)$ at $t = 0$, find the position of the particle at $t = 1$.

A. $\left\langle 1 - \frac{1}{e}, -2 \right\rangle$ B. $\left\langle 1 - \frac{1}{e}, 2\pi - 2 \right\rangle$ C. $\left\langle -\frac{1}{e}, 2\pi \right\rangle$ D. $\left\langle -1 - \frac{1}{e}, 2\pi - 2 \right\rangle$

Three Digit Answer:

6) (Calculator) $x(t) = -120\left(t^2 - 4t + e^{-t}\right)$ and $y(t) = 150\left(\sin(2t) - \cos t\right)$ represents the flight path of a small plane as shown by the figure to the right. Time t is measured in hours and x and y are measured in miles. The plane passes over a small island at $t = 1.43$ hours. At a certain point in time, the plane encounters bad weather and the pilot decides to turn back. The plane travels over the same island. What is the distance to the nearest mile that it travels between the time when it was first over the island and the time (to one decimal place) that it is next over the island?