

DIGITLE – BC CALCULUS

Puzzle 211 – Taylor Polynomials



Directions: The first 5 problems have single digit or letter answers. The 6th problem has a 3-digit answer (counting leading zeros if present). You have a choice: solve the easier single-character answer problems or tackle the more difficult 3-digit answer and the multiple choice. Once you have done that, attempt to solve the puzzle by entering the following url on your computer, tablet, or phone: <https://mastermathmentor.com/mmm/digitle.ashx>. The correct puzzle answer will be the digits/letters of your answer(s) scrambled. Use the following interpretation. You get 6 tries. Problems should be done without graphing calculators.

- Green :** the digit is in the answer and is in the correct spot. **Grey :** the digit is not in the answer.
Yellow: the digit is in the answer but is not in the correct spot.

Single Digit or Letter Answers:

1) Let f be a function having derivatives for all orders of real numbers. The first three derivatives of f at $x = 0$ are given in the table below. Use the third-degree Taylor polynomial at $x = 0$ to approximate $f\left(-\frac{1}{2}\right)$.

x	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$
0	4	-4	-6	12

2) The Taylor series for a function f converges to $f(x)$ for all x in the interval of convergence. The n th derivative of f at $x = 1$ is given by $f^{(n)}(x) = \frac{(-1)^n n!}{(n-1)^2}$ for $n \geq 2$. The graph of f has a relative minimum at the point $(2, 4)$. Use the 3rd degree Taylor polynomial for f , centered at $x = 2$ to approximate $f(2.2)$.

- A. 4.02 B. 4.068 C. 4.038 D. 4.042

3) Let $12P_4(x) = -6 + 3(x+1) + 18(x+1)^2 + (x+1)^4$ be the 4th degree Taylor polynomial for the function f about $x = -1$. What is the value of $f^{(4)}(-1)$?

4) A function has derivatives of all orders at $x = 0$. Let $P_n(x)$ denote the n th degree Taylor polynomial about $x = 0$. It is known that $f(0) = 0$, $P_1\left(-\frac{1}{2}\right) = \frac{-1}{16}$, and $P_2\left(\frac{-1}{2}\right) = \frac{1}{16}$. Find $f''(0)$.

5) Use the 2nd degree Taylor polynomial about $x = 0$ for $f(x) = e^{-x/3}$ to approximate $e^{-1/6}$.

- A. $\frac{85}{72}$ B. $\frac{13}{12}$ C. $\frac{31}{36}$ D. $\frac{61}{72}$

Three Digit Answer:

6) What is the coefficient for x^9 in the Taylor series for $\frac{2x}{2x+1}$ about $x = 0$?