# DIGITLE – BC CALCULUS
# Puzzle 213 – Error Bounds


**Directions**: The first 5 problems have single digit or letter answers. The 6th problem has a 3-digit answer (counting leading zeros if present). You have a choice: solve the easier single-character answer problems or tackle the more difficult 3-digit answer and the multiple choice. Once you have done that, attempt to solve the puzzle by entering the following url on your computer, tablet, or phone: https://mastermathmentor.com/mmm/digitle.ashx. The correct puzzle answer will be the digits/letters of your answer(s) scrambled. Use the following interpretation. You get 6 tries. Problems should be done without graphing calculators.

Green : the digit is in the answer and is in the correct spot. Grey : the digit is not in the answer.
Yellow: the digit is in the answer but is not in the correct spot.

1) How many terms will it take to approximate $\sum_{n=0}^{\infty} \frac{\cos n\pi}{3^n + n^3}$ with an error of less than 0.005?

2) Let $f$ be a function having derivatives of all orders for $x > 0$. Selected values of $f$ and its first three derivatives are indicated in the table. The function $f$ and these three derivatives are increasing on the interval $0 \le x \le 2$. Find the Lagrange error for $f(0.5)$ using the 2nd degree Taylor polynomial for $f$ about $x = 1$.

| $x$ | $f(x)$ | $f'(x)$ | $f''(x)$ | $f'''(x)$ |
|---|---|---|---|---|
| 0 | 8 | 4 | 12 | 48 |
| 1 | 12 | 16 | 32 | 96 |
| 2 | 16 | 32 | 60 | 144 |

3) Use the 3rd degree Maclaurin Polynomial for $\sin x$ and its Lagrange error to make a statement about $\sin 1$.

A. $\frac{19}{24} < \sin 1 < \frac{7}{8}$ B. $\frac{33}{40} < \sin 1 < \frac{101}{120}$ C. $\frac{5}{12} < \sin 1 < \frac{11}{12}$ D. $\frac{7}{15} < \sin 1 < \frac{13}{15}$

4) Find the Lagrange error in calculating $f(0.5)$ for the 3rd degree Taylor polynomial for $\frac{8x}{e^x}$ about $x = 0$.

A. $\frac{1}{96}$ B. $\frac{1}{64}$ C. $\frac{1}{48}$ D. $\frac{1}{2}$

5) Let $f$ be a function having derivatives for all orders of real numbers. The function and its first three derivatives at $x = 0$ are given in the table. The 4th derivative of $f$ satisfies the inequality $\left|f^{(4)}(x)\right| \le 32$ for all $x$ in the interval [0, 2]. Find the maximum value of $f\left(\frac{3}{2}\right)$.

| $x$ | $f(x)$ | $f'(x)$ | $f''(x)$ | $f'''(x)$ |
|---|---|---|---|---|
| 0 | 2.5 | 4 | –6 | –8 |

**Three Digit Answer:**

6) How many terms are necessary to approximate $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$ with an error of less than or equal to 0.04?