Waiting for Dorian

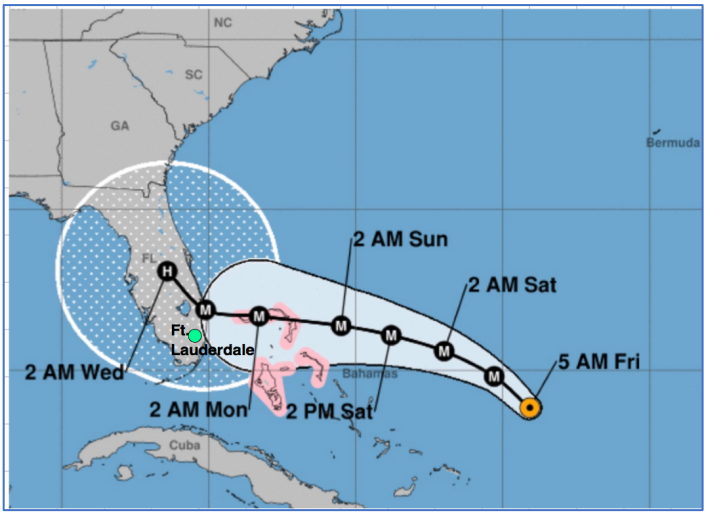
Stu Schwartz – MasterMathMentor.com

It is Monday, September 2, 2019. My Mom lives in South Florida, just north of Ft. Lauderdale. The news down there for the last week has been the path of devastating Hurricane Dorian. I follow the updates religiously and for a while, they were really dire. As I write this, my Mom is hunkered down waiting to feel the effects of the hurricane. And very worried. If she is worried, so am I as she lives alone. Very luckily, conditions are presently not expected to be bad in her area. However, the mathematics of this is fascinating to me and to get my mind back into math (where I am most comfortable), I decided to write about it.

The Weather Channel

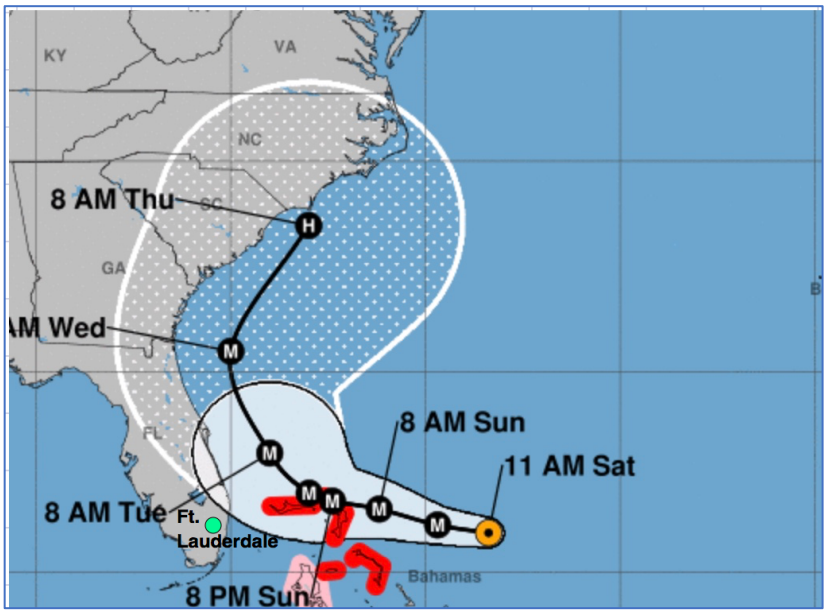
So many people think that a hurricane is a point. It isn't. It is huge. The picture below gives you an idea of the size of a hurricane. Your first thought might be to escape: Escape where? From the east coast, driving to the west coast, a mere 100 miles makes little difference. Driving north with everyone else trying to do the same thing still leaves you in the possible path of the hurricane.
For most people, they have to hunker down. Luckily, since Florida has weathered many hurricanes, officials are incredibly well organized and have plans that they can immediately put into effect. 13,000 people working with Florida Power and Light are mobilized and moved into locations in order to get power restored as soon as they can. There are shelters open, some specifically for older people and those that are pet-friendly. Markets are immediately stocked with water and supplies and are open as long as possible, and open as soon as the danger passes. Living through a hurricane threat is not fun but there is no place I'd rather be than in Florida if I had to face the prospect of one.

At 5 AM on Friday, August 30, while the storm was still about 700 miles away from South Florida, the eye of the storm where the projected Category 5 (157 mph and greater winds are) was projected to come in just above Ft. Lauderdale. But this was still very uncertain. The "cone of concern" (the areas that the storm will be at with 2/3 probability) encompasses all the area from the southern tip to well up the coast within 3 days. After that time, the cone stretches all the way to the Florida Keys into Georgia. But if you follow the center path of the storm based on predictions, it would be a devastating hit to the state (M refers to a major hurricane, category 3 or greater, H as a category 1 or 2 hurricane).



Courtesy of National Hurricane Center

After what to me felt like an eternity but was only 2 days, the track changed as the hurricane slowed down (and intensified). Below was the projected track as of 11 AM on Saturday, August 30. Notice that Ft. Lauderdale is just out of the cone of concern. That does not mean that we wouldn't get tropical force winds as a hurricane is not a point but a huge circle with diameter around 300 miles. But, Floridians are used to that. Still, no one has experienced 180 mph winds at the center of the hurricane and half of Florida was in the cone of concern.



Courtesy of National Hurricane Center

I was talking to a friend of hers who was unconcerned. The eye of Dorian was directly east of Ft. Lauderdale, about 500 miles away. It was going north (hurricanes rarely go south) but gaining latitude very slowly). So, it was close to impossible, he reasoned, for the center of the storm to hit us.

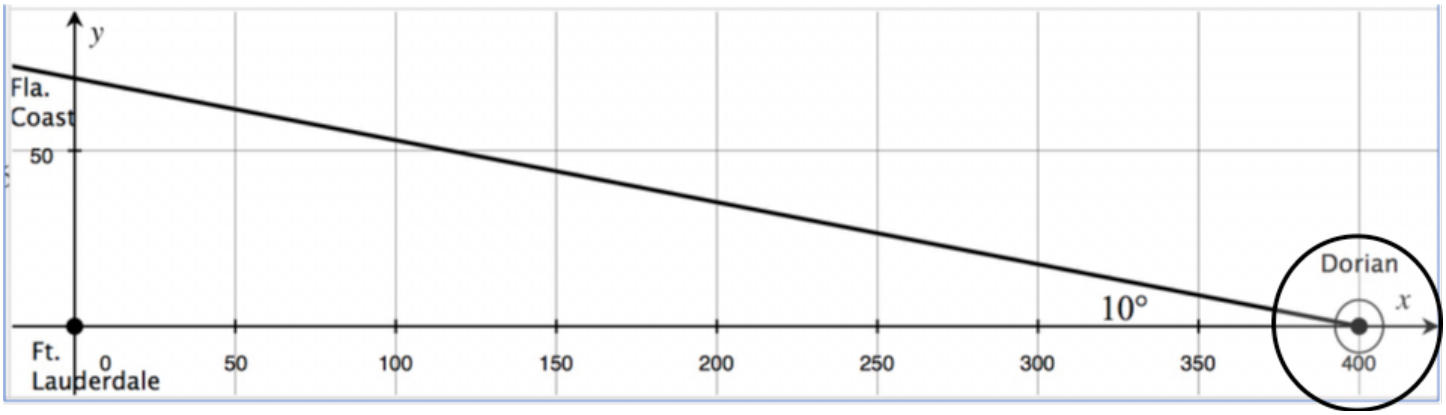
But it is not that easy. First, the cone of concern is manmade: it represents something that occurs 2 out of 3 times. It is still all about probability. And there were some simulation tracks that said that Dorian might actually move a little southwest. Prediction is not an exact science.

Still, the further north that the storm tracked, albeit slowly, meant that it was still coming towards us. The distance between the eye of the hurricane and Ft. Lauderdale was decreasing. When, I wondered, would the hurricane be closest to her and give her the highest winds? Aha, Calculus!

So, to get my mind off possible impending doom, I decided to create a calculus problem based on this situation. The numbers aren't exact to the real situation but fairly close. It involves a little BC calculus but mostly AB calculus and precalculus.

The problem:

Dorian is 400 miles due east of Ft. Lauderdale. It is traveling at 8 mph on a course of 280°. I assume that the Florida coastline is vertical (runs north-south). Also, the eye of the hurricane we will assume to be a circle 15 miles wide. Again, the winds around the eye (the eyewall) is where the hurricane is most destructive with winds over 157 mph in a category 5 hurricane. There is no category 6 but Dorian would certainly be one. Hurricane winds extend about 45 miles from the center. The figure below describes the initial position. The smaller circle represents the eyewall and the larger circle represents the area of hurricane force winds.



The question is: What is the closest the eyewall will be to Ft. Lauderdale and how long will that take?

The analysis:

This initially is a vector problem (parametric equations). The velocity of Dorian is $\langle 8\cos 280°, 8\sin 280° \rangle$. However, while conventional directions assume that $0°$ is due north, $90°$ is due east, $180°$ is due south and $270°$ is due west, our calculators assume that $0°$ is due east, $90°$ is due north, $180°$ is due west and $270°$ is due south. Since we will be using calculators, we will write the velocity of Dorian as $\langle 8\cos 170°, 8\sin 170° \rangle$. And since we need to be working in radians, we will write v = the velocity of Dorian to be $\left\langle 8\cos\frac{17\pi}{18}, 8\sin\frac{17\pi}{18} \right\rangle$.

The initial position of the center of Dorian is the point (400,0) and Ft. Lauderdale is (0,0) with x and y in miles.

We need to write a position vector $s(t)$ for the center of Dorian. Since the derivative of position is the velocity, then the integral of velocity is the position.

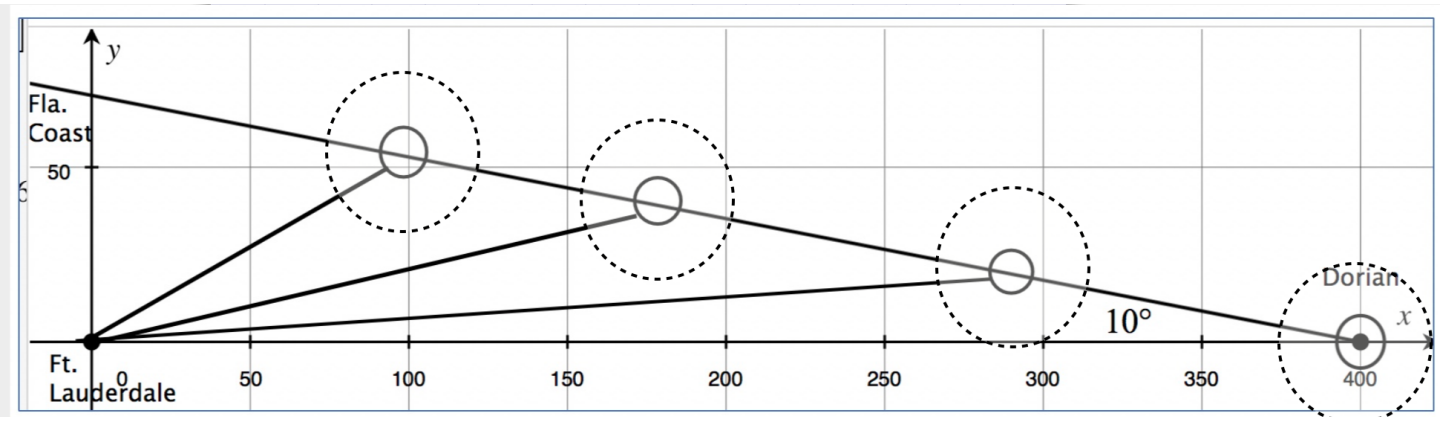
$$\int 8\cos\frac{17\pi}{18}dt = 8t\cos\frac{17\pi}{18} + C_1 \qquad \int 8\sin\frac{17\pi}{18}dt = 8t\sin\frac{17\pi}{18} + C_2$$

(remember that $\cos\frac{17\pi}{18}$ and $\sin\frac{17\pi}{18}$ are constants).

To find our constants, we use the initial point (400, 0).

$$8(0)\cos\frac{17\pi}{18} + C_1 = 400 \Rightarrow C_1 = 400$$
$$8(0)\sin\frac{17\pi}{18} + C_2 = 0 \Rightarrow C_2 = 0$$
$$s(t) = \left\langle 8t\cos\frac{17\pi}{18} + 400, 8t\sin\frac{17\pi}{18} \right\rangle$$

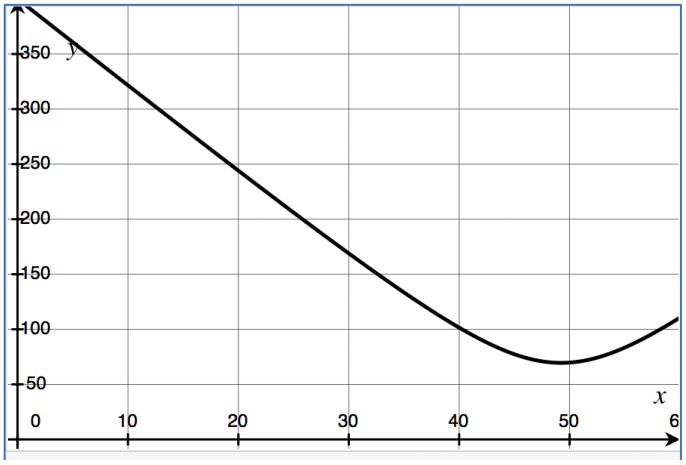
Plotting that vector, we can see that the distance between Ft. Lauderdale and the eyewall is shrinking. But damaging hurricane force winds can occur well outside the eyewall. The velocity of the winds from the eyewall is a function that decreases over distance. As stated in this problem, we are assuming that hurricane force winds (93 mph and greater) extend 45 miles from the center of the storm). It's not a matter of eyewall is bad and anything else is good.



So, we need to find the distance from Ft. Lauderdale at (0, 0) to any point (x, y) on the eye (we are interested in the eyewall, but it is much easier to use the eye, the exact center of the hurricane. The lines I drew above don't go through the exact center of the eye, but the distance is negligible. Using the distance formula, we get

$$d = \sqrt{\left(8x\cos\frac{17\pi}{18} + 400\right)^2 + \left(8x\sin\frac{17\pi}{18}\right)^2}$$

Let's plot that curve.



It appears that the distance from the center of Dorian to Ft. Lauderdale is certainly decreasing. Since the graph appears somewhat linear at first, it looks like the distance from the center of Dorian to Ft. Lauderdale is approximately $\frac{d(20)-d(0)}{20-0} = \frac{244-400}{20} = 7.8$ miles per hour or about 187 miles per day.

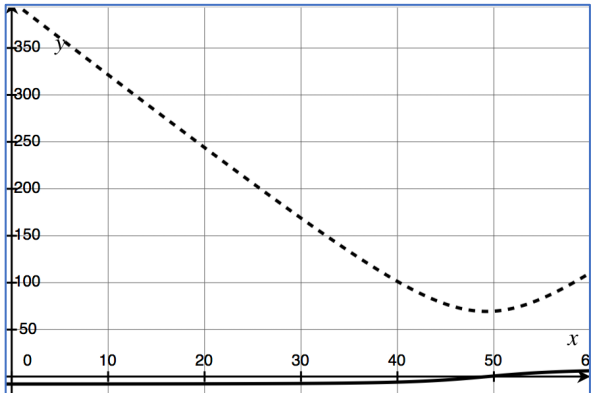
We could use the calculator to find the number of hours where the distance from the eye to Ft. Lauderdale is a minimum. But this is a calculus problem so let's do some complicated derivative analysis. Take the derivative and set it equal to zero.

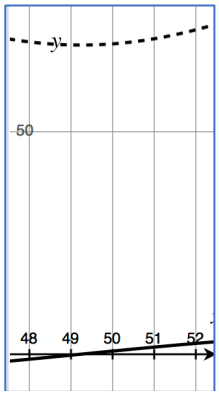
$$s=\sqrt{\left(8x\cos\frac{17\pi}{18}+400\right)^2+\left(8x\sin\frac{17\pi}{18}\right)^2}$$

$$\frac{ds}{dt}=\left(\frac{1}{2}\right)\frac{2\left(8x\cos\frac{17\pi}{18}+400\right)\left(8\cos\frac{17\pi}{18}\right)+2\left(8x\sin\frac{17\pi}{18}\right)\left(8\sin\frac{17\pi}{18}\right)}{\sqrt{\left(8x\cos\frac{17\pi}{18}+400\right)^2+\left(8x\sin\frac{17\pi}{18}\right)^2}}$$

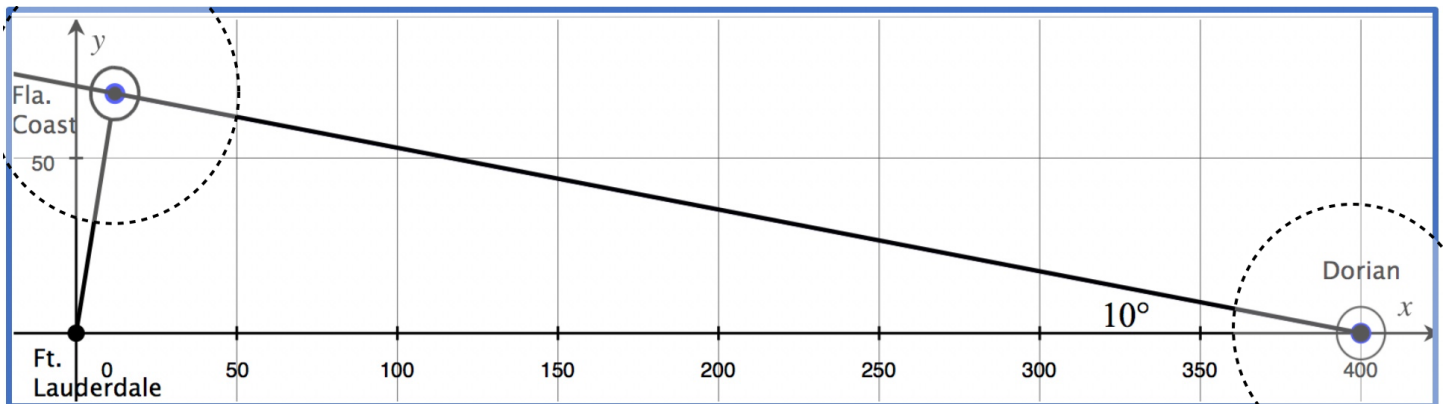
$$0=\frac{\left(8x\cos\frac{17\pi}{18}+400\right)\left(8\cos\frac{17\pi}{18}\right)+64x\sin^2\frac{17\pi}{18}}{\sqrt{\left(8x\cos\frac{17\pi}{18}+400\right)^2+\left(8x\sin\frac{17\pi}{18}\right)^2}}$$

Graphing this mess, we see the distance function from Ft. Lauderdale to Dorian in a dashed curve and its derivative in a solid curve. However, the derivative is tough to see because the distance From Dorian to Ft. Lauderdale is decreasing very slowly (its derivative is just slightly negative and fairly constant over the first 40 hours). Zooming in to the location where the distance function is a minimum and its derivative equals zero, we get $x = 49.24$ in both cases.





Since we substituted x for t for graphing purposes, we determine that in $t = 49.24$ hours, just over 2 days, the eye of the hurricane will make its closest approach to Ft. Lauderdale as shown by the figure below. After that, the distance will be increasing.



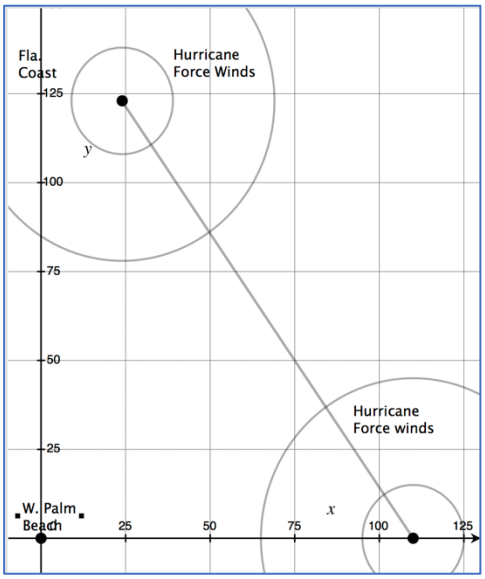
At $t = 49.24$ hours, $d = \sqrt{\left(8(49.24)\cos\frac{17\pi}{18} + 400\right)^2 + \left(8(49.24)\sin\frac{17\pi}{18}\right)^2} = 69.46$ miles. That represents the distance between Ft. Lauderdale and the eye. Subtracting 7.5 miles for the radius of the eyewall, the conclusion is that the devastating winds of Dorian will be about 62 miles at closest approach to Ft. Lauderdale if the path remained the same. Since hurricane force winds extend 45 miles from the center, Ft. Lauderdale would be about 24 miles from being exposed to hurricane force winds.

That is why Ft. Lauderdale was no longer in the cone of concern. And that was based on the assumption that Dorian would not turn south, even one degree, an assumption on which the forecasters had great confidence. Still, typical hurricanes have a radius of 150 miles so 62 miles is well inside tropical storm winds which can be damaging.

Of course, this didn't come to pass. It took days and days for Dorian to travel towards the Florida coast from that initial position because it slowed down. That slowing down appears to be the savior for Ft. Lauderdale and South Florida as it allowed a blocking mass of high pressure to break down and allowed Dorian an escape route to the north. That slowing down (and intensifying because of the warmth of the water) though was devastating to the Bahamas as 200 mph winds were blowing for pretty much a day over one small area. Consider: An F3 devasting tornado has a diameter of about 400 yards with about 200 mph winds. Its funnel cloud moves at a rate between 10 and 50 mph. Compare that to Dorian whose diameter is 15 miles traveling at 2 mph. It would be like having a tornado over your house for 24 hours.

Now all I have to do is to wait to see if my Mom got any effects from the storm. Watching and waiting …

Fast forward: It is now a day later and most of the danger has passed for her. The hurricane is 110 miles east of West Palm moving in the direction of $325°$, albeit at 1 mph. Hurricane force winds extend 45 miles from the center. The first predicted and anticipated turn has begun. The maximum wind speed has dropped to 120 mph, still dangerous, but no longer the catastrophic velocities of the previous days. Dorian spinning essentially motionless sucked the heat out of the water below, robbing it of fuel. But is there still danger for the rest of Florida?



The figure to the right shows the path of Dorian over the next 150 hours on that course. West Palm appears out of the danger for hurricane force winds but still is subject to tropical force winds that extend 160 miles from the center.

The eye of the hurricane is getting very close to the Florida coast. However, the hurricane is expected to speed up, bringing it close to the coast much more quickly. It is expected to turn to the north. The question is when. That will play out in the next day or so. If it does, it will hug the coast, hopefully staying at least about 50 miles from the coast. At this writing, that appears to be occurring. When the east coast turns to the northeast in Georgia, the hurricane is expected to do the same. Hopefully it will.

As a math teacher, I have unbelievable respect for the meteorologists and the experts at the National Hurricane Center. To create a forecast with extreme accuracy with all the variable factors such as the steering winds, interaction with land masses, water temperature, and areas of high and low pressure which all of which will determine the path and strength of the hurricane is incredibly hard. Add to the fact that we are talking a life-and-death situation and the decisions that normal people ultimately make is based on these predictions. These are amazing people and they don't get the accolades they richly deserve.