READING MATH FOR THE BLIND AND DYSLEXIC

Years ago, I read and record for the blind and dyslexic. I did this at Learning Ally (http://www.learningally.org/) in King of Prussia, Pa. The mission of this organization is providing books of all kinds to students with a "documented print disability."

From the Learning Ally website:

> Founded in 1948 as Recording for the Blind, Learning Ally serves more than 300,000 K-12, college and graduate students, veterans and lifelong learners – all of whom cannot read standard print due to blindness, visual impairment, dyslexia, or other learning disabilities. Learning Ally's collection of more than 75,000 digitally recorded textbooks and literature titles – downloadable and accessible on mainstream as well as specialized assistive technology devices – is the largest of its kind in the world. More than 6,000 volunteers across the U.S. help to record and process the educational materials, which students rely on to achieve academic and professional success.
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> Learning Ally, a 501(c)3 nonprofit, is funded by grants, state and local education programs, and the generous contributions of individuals, foundations and corporations.

At the studio, I read and record educational textbooks. At first, I learned to read general texts – from psychology to history, from cooking to physical education, but quickly I learned to become a specialty reader. And of course, my specialty is secondary mathematics.

Reading mathematics is not easy. As teachers, we read math from the board or a book out loud to the class all the time. But what about the students who are blind? How in the world can they comprehend this material, especially when it is complex?

For instance, try reading the equation $4(x+2)-2x=\frac{5-x}{3}$.

You would probably say: Four times the quantity *x* plus two minus 2*x* equals 5 minus *x* over 3.

And for students who are sighted, this creates no problem. But suppose you did not have the advantage of having this equation in front of you? Couldn't your words be interpreted as: $4(x+2-2x)=5-\frac{x}{3}$ or some other interpretation? A student who was blind would misinterpret your words through no fault of his own.

Reading math requires some special knowledge and learning some conventions: For instance, the way I would read the equation: $4(x+2)-2x=\frac{5-x}{3}$ is:

> Four times the quantity *x* plus two - end quantity - minus two *x* equals five minus *x* all over three.

There would be an assumption that students understand that a quantity means using parentheses. Also assumed is that the student who is using this book is learning in class and listens to his teacher use proper math terminology. However if $4(x+2)$ was to be read before the concept of expressing this was formally taught, it would be read:

> Four left parentheses *x* plus two right parentheses.

A simple factoring example like $x^2-4=(x+2)(x-2)$ might be read as:

> *x*-squared minus four equals quantity *x* plus two times quantity *x* minus 2.

But that could be interpreted as $x^2-4=x+2(x-2)$ which of course is not equivalent to the example. So the correct reading would have to be:

> *x*-squared minus four equals quantity *x* plus two - end quantity times quantity *x* minus 2.

It's like having to unlearn all the ways I have ever spoken math to my classes!

If you heard the words: The sine of *e* to the two *x* plus one equals the square root of *x* minus two plus *k*, would you write:

$\sin(e^{2x}+1)=\sqrt{x-2}+k,$

$\sin(e^{2x})+1=\sqrt{x-2+k}$

or some other variation?

Certainly the words are ambiguous. We can listen for pauses:

> The square root of *x* minus two.
> The square root of *x* (PAUSE) minus two.

But how long do we pause? Clearly, pausing is not a good solution so some conventions must be learned to adequately describe this equation to someone who is blind. Students who are dyslexic are another matter. I am curious if someone who is dyslexic (but has the equations in front of them) would benefit from hearing the equations read. I just don't know. But when we read, we have no idea whether the students listening are dyslexic (the majority) or blind.

The proper way to read $\sin(e^{2x})+1=\sqrt{x-2}+k$ is:

> The sine of *e* to the two *x* power plus 1 equals the square root of *x* minus two end square root plus *k*.

That's a lot to say!

A typical text has numerous equations like this in the description, exercises, and solutions (yes, we read them too). So decisions about how to read these must be made on the fly. It is very challenging.

There is a "Bible" created by Lawrence Change called Larry's Speakeasy which gives general conventions for reading math, from basic arithmetic to the most advanced calculus and beyond. It is interesting reading (for math geeks!):

https://02522-cua.github.io/lecturenotes/pdf/Chang_1983_Handbook%20for%20Spoken%20Mathematics.pdf

I know little about dyslexia but in my last few years of teaching, I found that more and more students were diagnosed with it.

From Wikipedia:

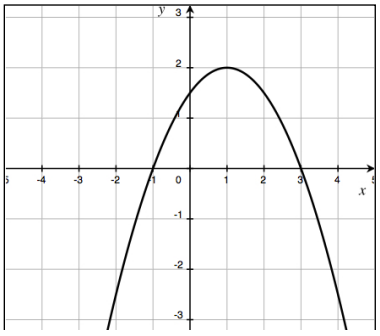
> Some early symptoms that correlate with a later diagnosis of dyslexia include delays in speech, letter reversal or mirror writing and being easily distracted by background noise. At later ages symptoms can include a difficulty identifying or generating rhyming words, or counting syllables in words (a difficulty segmenting words into individual sounds, or blending sounds to make words, a difficulty with word retrieval or naming problems, commonly very poor spelling which has been called dysorthographia (orthographic coding), and tendencies to omit or add letters or words when writing and reading are considered classic signs. Other classic signs for teenagers and adults with dyslexia include trouble with summarizing a story, memorizing, reading aloud, and learning a foreign language A common misconception about dyslexia is that dyslexic readers write words backwards or move letters around when reading – this only occurs in a very small population of dyslexic readers. Dyslexic people are better identified by writing that does not seem to match their level of intelligence from prior observations.

I know that there were programs in the school available to help those students identified with dyslexia. But dyslexia was, at least how I interpreted it, a problem with reading and confined to the English class. But with the great number of word problems that permeate typical math curriculum as well as state tests, dyslexia is certainly a deterrent to math education. Add to that, omitting, transposing, or deleting digits in mathematics is suicidal!

I wish I knew more about math dyslexia but there just isn't that much information available or even known. The type of math dyslexia I refer to is not the same as dyscalculia which is a learning disability involving innate difficulty in learning or comprehending arithmetic.

At Learning Ally, we read textbooks that have been requested by its members. I am amazed at not only how many math books we read but their complexity. I have read basic algebra and geometry books as well as advanced calculus and statistics texts. I wish I knew whether these books were for blind students or dyslexic students (because I might read them differently), but for whoever has requested them to be read, hopefully these students are being helped.

Graphs must also be described. The way I would describe the figure below is:



> There is a graph with *x* running from negative four to four with a scale of one and *y* running from negative three to three with a scale of one. There is a parabola that opens downward whose vertex is the point one – two and the parabola goes through the points negative one – zero and three – zero.

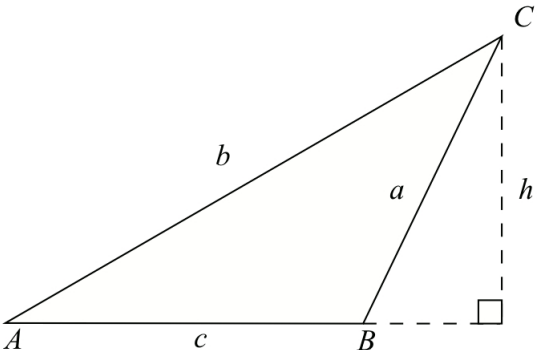
If the figure appears in the book before parabolas are formally taught, that adds an additional challenge: Students then don't understand what a parabola looks like so it must be described:

There is a graph with x running from negative four to four with a scale of one and y running from negative three to three with a scale of one. The graph is a curve that comes up from the third quadrant hitting the x-axis at x equals negative one. It hits the y-axis at approximately the point zero – one point five and then hits its peak at the point one – two before going back down, hitting the x-axis at x equals three before going down steeply into the fourth quadrant.

I might be tempted to use the word "arch" but I have no idea whether blind students understand and/or can visualize an arch. Again, these graphs are described quickly and on the fly. There is no one right way to describe it – but it should be as clear as possible to the listener.

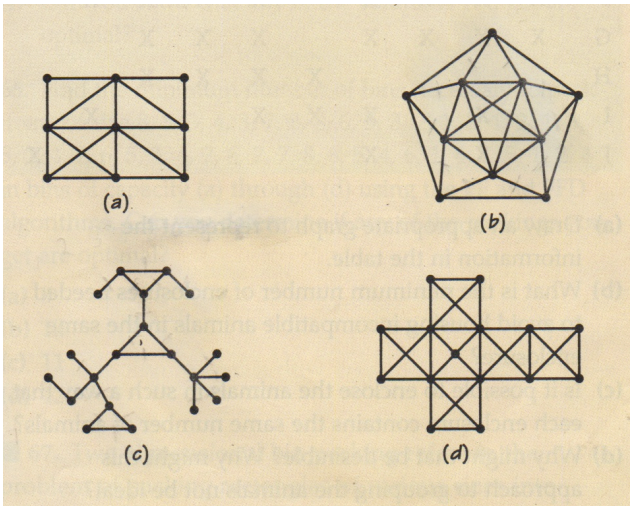
Add asymptotes, horizontal tangents, points of inflection, and repeating patterns and these graphs can become difficult to explain. Whether a dyslexic person needs these explanations, I have no idea and how a blind person can understand the concept of a parabola or an inflection point is a source of wonder within me. But the fact that we read so many math textbooks tells me that they must be useful to the people who have requested them.

Geometry can be especially tough. Assuming that students knew what angles, triangles, and perpendicular means, the figure below would be described like this:

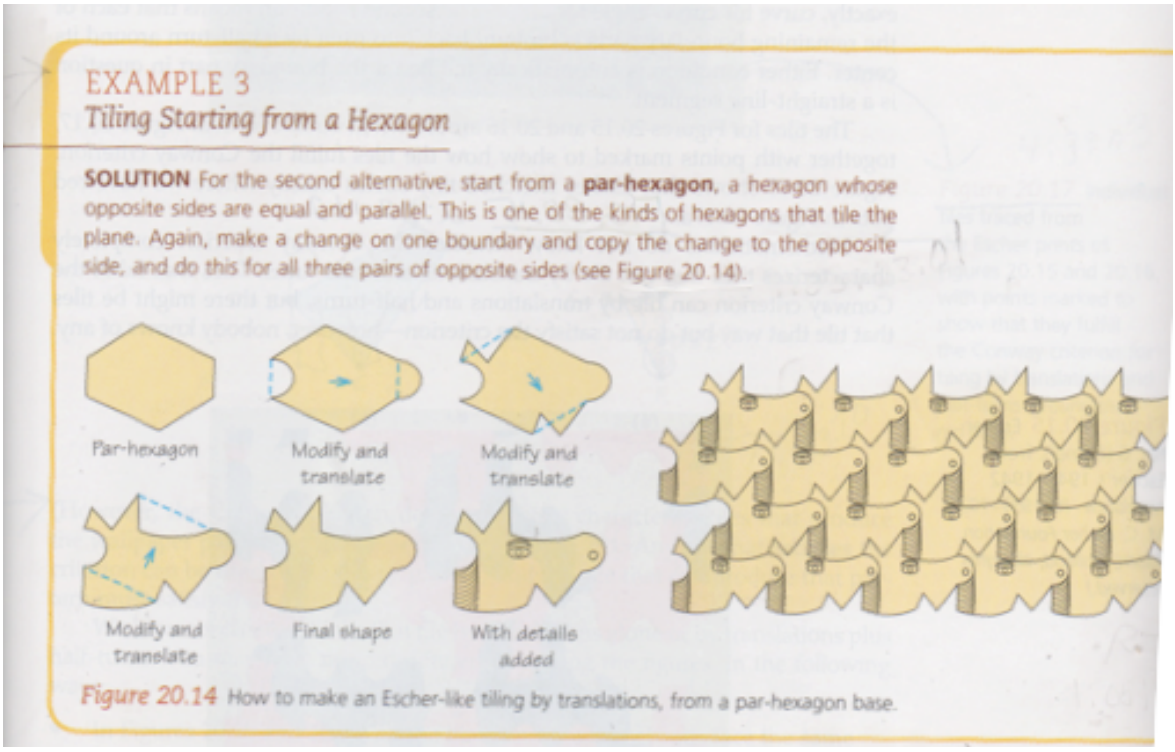


There is a figure showing a triangle caps ABC. Caps AB are on the horizontal and caps C is above and to the right of caps B. The side opposite angle cap A is a, the side opposite angle cap B is b and the side opposite angle cap C is c. The line caps AB is extended horizontally to the right with a dashed line. There is also a dashed line labeled h that comes vertically down from cap C and is perpendicular to the extension line from caps AB.

And that's an easy one. Think about how you would describe these figures to someone. Try describing them to your students and see if they can draw these figures.



And then we sometimes get to describe figures like the one below. I would have to stop the recording and think about what I was going to say. Still, I wonder if anyone, blind or no, could draw these figures based on my description. And if he can, it says more about the listener than it does about me.



EXAMPLE 3

Tiling Starting from a Hexagon

SOLUTION For the second alternative, start from a **par-hexagon**, a hexagon whose opposite sides are equal and parallel. This is one of the kinds of hexagons that tile the plane. Again, make a change on one boundary and copy the change to the opposite side, and do this for all three pairs of opposite sides (see Figure 20.14).

Figure 20.14 How to make an Escher-like tiling by translations, from a par-hexagon base.

There used to be Learning Ally locations in Arizona, California, Colorado, Florida, Georgia, Illinois, Massachusetts, New Jersey, New York, Pennsylvania, Texas, Tennessee, and Virginia as well as Washington, DC. However, because of better technology, reading can be done from home. Specialty readers are especially needed because of their intimate knowledge of the subjects at hand. They aren't limited to the sciences. Art, music, languages, and elementary education, are just a few of the specialty areas in which reading is needed and requested.

It isn't one-to-one teaching but it is nice to know that there is another way to have an impact on the education of our students beyond the regular classroom.

If anyone would like additional information on how your school can become a member of Learning Ally, check out their website: http://www.learningally.org/ .