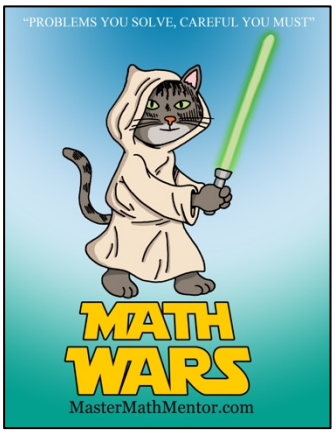
Math Wars – AB Calculus
Scrambled 158 – Limits & Derivatives



Maximum Time: 8 Minutes

1. (1 pt) A particle is moving along the *y*-axis with position $y(t) = -6t^3 - t^2 + 10t - 4$. Describe the behavior of the particle at $t = 1$ second.

A. Above the *x*-axis and speeding up
B. Above the *x*-axis and slowing down
C. Below the *x*-axis and speeding up
D. Below the *x*-axis and slowing down

C. $y(t) = -6t^3 - t^2 + 10t - 4 \Rightarrow y(1) = -6 - 1 + 10 - 4 < 0$ below *x*-axis
$v(t) = -18t^2 - 2t + 10 \Rightarrow v(1) = -18 - 2 + 10 < 0$
$a(t) = -36t - 2 \Rightarrow a(1) = -36 - 2 < 0$ Since $v(1)$ and $a(1) < 0$, particle speeding up

2. (3 pts) Let $f(x) = (x^2 - 16)(x^2 - 4)$. At how many value of *c* does Rolle's Theorem hold on [–4, 2] ?

A. 3 B. 2 C. 1 D. 0

B. $f(-4) = f(2) = 0$
$f(x) = x^4 - 20x^2 + 64 \Rightarrow f'(c) = 4c^3 - 40c = 0$
$4c^2(c^2 - 10) = 0 \Rightarrow c = 0, \pm\sqrt{10}$ Only $-\sqrt{10}, 0$ are on $[-4, 2]$

3. (5 pts) Find the slope to the curve $xy^2 + x^2y = 8x$ at $(-2, 4)$.

A. $\frac{-1}{4}$ B. $\frac{-2}{3}$ C. $\frac{2}{5}$ D. 2

B. $2xy\frac{dy}{dx} + y^2 + x^2\frac{dy}{dx} + 2xy = 8$
$-16\frac{dy}{dx} + 16 + 4\frac{dy}{dx} - 16 = 8$
$-12\frac{dy}{dx} = 8 \Rightarrow \frac{dy}{dx} = \frac{-2}{3}$

4. (7 pts) If $f(x)=\dfrac{x-4}{x^3-64}$, find $f'(x)$

A. $\dfrac{-2x}{\left(x^2-16\right)^2}$ B. $\dfrac{-2x-4}{\left(x^2+4x+16\right)^2}$ C. $\dfrac{1}{2x+4}$ D. $\dfrac{1}{3x^2}$

B.
$$f(x)=\frac{x-4}{(x-4)\left(x^2+4x+16\right)}=\frac{1}{x^2+4x+16}$$
$$f'(x)=\frac{-2x-4}{\left(x^2+4x+16\right)^2}$$

5. (9 pts) Let $f(x)=1000\sin\left(\dfrac{1}{2}x\right)$. What is the first value of n for which the nth derivative of f at $x=3\pi$ is greater than 0 but less than 1?

A. 9 B. 10 C. 11 D. 12

$$f'(x)=1000\left(\frac{1}{2}\right)\cos\left(\frac{1}{2}x\right)\Rightarrow f'(3\pi)=0$$
$$f''(x)=-1000\left(\frac{1}{2}\right)^2\sin\left(\frac{1}{2}x\right)\Rightarrow f''(3\pi)=\frac{1000}{4}$$
$$f'''(x)=-1000\left(\frac{1}{2}\right)^3\cos\left(\frac{1}{2}x\right)\Rightarrow f'(3\pi)=0$$

B. $$f^{(4)}(x)=1000\left(\frac{1}{2}\right)^4\sin\left(\frac{1}{2}x\right)\Rightarrow f'(3\pi)=\frac{-1000}{16}$$

The odd derivatives will be zero and the 4th, 8th, 120th... derivatives are negative

$$f^{(6)}(x)=-1000\left(\frac{1}{2}\right)^6\sin\left(\frac{1}{2}x\right)\Rightarrow f'(3\pi)=\frac{1000}{64}$$
$$f^{(10)}(x)=-1000\left(\frac{1}{2}\right)^{10}\sin\left(\frac{1}{2}x\right)\Rightarrow f'(3\pi)=\frac{1000}{1024}$$