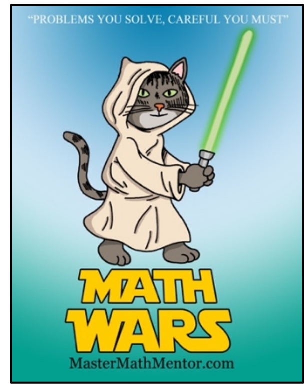
Math Wars – BC Calculus
Topic 216 – Integral and Comparison Tests



Maximum Time: 7 Minutes

1. (1 pts) Let $f(x)$ be a positive, continuous, decreasing function. Suppose $\int_{1}^{\infty} f(x)\,dx = 100$. Which of the following must be true about $\sum_{n=1}^{\infty} f(n)$?

A. it converges
B. it converges and is greater than 100
C. it converges and is less than 100
D. it diverges

2. (3 pts) We are given that $f(x) > 0$ for all x. Suppose $\sum_{n=1}^{\infty} \frac{1}{f(n)}$ converges. If a is a positive constant, consider the convergence of the following three series. Choose the most accurate answer.

I. $\sum_{n=1}^{\infty} \frac{a}{f(n)}$ II. $\sum_{n=1}^{\infty} \frac{1}{f(n)+a}$ III. $\sum_{n=1}^{\infty} \frac{1}{f(n)-a}$

A. I, II and III must all converge
B. I and II must converge, III could converge
C. I must converge, II and III could converge
D. I, II and III could converge

3. (5 pts) Using the limit comparison test, which of the following converges?

I. $\sum_{n=1}^{\infty} \frac{n}{n-\cos n}$ II. $\sum_{n=1}^{\infty} \frac{n}{n^2-\cos^2 n}$ III. $\sum_{n=1}^{\infty} \frac{n}{n^3-\cos^3 n}$

A. I only B. II only C. III only D. II and III only

4. (7 pts) When determining the possible convergence of $\sum_{n=1}^{\infty} \tan\frac{1}{n}$, the limit comparison test is used, comparing the series to $\sum_{n=1}^{\infty} \frac{1}{n}$. Because the result of the comparison is ________, the conclusion is that the series $\sum_{n=1}^{\infty} \tan\frac{1}{n}$ is ___________

A. 0, convergent B. 0, divergent C. 1, convergent D. 1, divergent

5. (9 pts) Using the Integral test on $\sum_{n=4}^{\infty} \frac{2}{n^2-4n+3}$, the value of the integral is _________, leading to a conclusion that the series is ___________.

A. $\ln\frac{1}{3}$, convergent B. ln 3, convergent C. $\ln\frac{1}{3}$, divergent D. ∞, divergent