



# MasterMathMentor.com

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## Super Free-Response Practice AB Question 1

**A graphing calculator is needed for this problem.**

**It is recommended that you take no more than 40 minutes for this problem.**

1. A new shopping website called “Automaton” generates sales (orders) at a rate modeled by the function  $S$  which is given by  $S(t) = 35 - 25\cos\left(2 + \frac{\pi t}{12}\right)$ . Orders are packed for shipping at a rate modeled by the function  $P$  given by  $P(t) = 35 - t + e^{0.014t^2}$ . Both  $S(t)$  and  $P(t)$  are measured in thousands per hour and  $t$  is measured in hours since 8 AM ( $t = 0$ ). The models are valid for 8 AM until 12 midnight. At 8 AM, there are 2,500 orders that not been packed.

- (a) Find the values of  $S(5)$  and  $S'(5)$  and explain their meaning using proper units.

Your Score \_\_\_\_\_

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- (b) At approximately what time (to the nearest 20 minutes) is the sales rate a maximum? Justify your answer.

Your Score \_\_\_\_\_

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- (c) At approximately what time (to the nearest 20 minutes) is the packing rate a minimum? Justify your answer.

Your Score \_\_\_\_\_

(d) Are there any time periods when both the sales rate and the packing rate are decreasing? Explain.

Your Score \_\_\_\_\_

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(e) How many sales are generated over the entire modeling period? Give your answer to the nearest whole number.

Your Score \_\_\_\_\_

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(f) Approximate the number of orders that were packed over the modeling period using a trapezoidal rule using data 4 hours apart. Show how you arrived at your answer. Give your answer to the nearest whole number.

Your Score \_\_\_\_\_

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(g) What is the average number of sales per hour that are generated over the modeling period?

Your Score \_\_\_\_\_

(h) For 50% of the modeling period, sales per hour will be greater than the average sales per hour. Decide if you agree with this statement and explain.

Your Score \_\_\_\_\_

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(i) When the sales per hour exceeds 50,000, a secondary website is opened to handle the excess sales. How many of these excess sales are there during the modeling period?

Your Score \_\_\_\_\_

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(j) When the packing rate exceeds 33,000 orders per hour, extra packers need to be hired. What is the average number of orders per hour to be packed during these time periods?

Your Score \_\_\_\_\_

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(k) How many orders are still to be packed at 12 noon?

Your Score \_\_\_\_\_

(l) Is this number of orders yet to be packed at 12 noon increasing or decreasing at that time? Explain.

Your Score \_\_\_\_\_

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(m) At what time of the day (to the nearest 20 minutes) is the greatest number of orders to be packed a maximum? Justify your answer.

Your Score \_\_\_\_\_

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(n) What is the greatest number of orders to be packed during the modeling period?

Your Score \_\_\_\_\_

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(o) Suppose that trucks show up to the packing facility every 3 hours and take all orders that were packed to a mailing facility. How many orders are waiting to be taken away at 4:30 PM?

- (p) There is a time of day represented by  $t = k$  such that the number of sales prior to  $k$  is the same as the number of sales after  $k$ . Write but do not solve an equation that uses  $k$ .

Your Score \_\_\_\_\_

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- (q) Suppose the website closes for maintenance at 12 midnight, so no orders are taken overnight. If the packers can pack 30,000 orders per hour, approximately what time (nearest 20 minutes) will all the orders be packaged?

Your Score \_\_\_\_\_

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### Grading:

Grade yourself according to the rubric that either your teacher gave you or you can find in the [MasterMathMentor.com](http://MasterMathMentor.com) website. Be strict with yourself. Additional grading tips are given in the accompanying YouTube video for this problem.

Section	Pts available	Your score
a	3	
b	3	
c	3	
d	2	
e	3	
f	3	
g	2	
h	2	
i	3	

Section	Pts available	Your score
j	4	
k	4	
l	2	
m	4	
n	3	
o	2	
p	1	
q	3	
<b>Total</b>	47	

There are 47 points available for this question. There is no exact formula for what number of points constitutes a 5, 4, 3, 2, or 1 on the A.P. Exam. However, these percentages are what have been used in the past based on exams released by the College Board. While you can extrapolate for just this question, realize that it tests only a limited number of AP topics. It is recommended that you do a number of questions in this series, combine your results, total your points, and then use these percentages to get a feel for how you will do in the AP exam, and more importantly, what concepts you need to strength to improve your score.

Grade	Percentage	This Question
5	70%	33 – 47
4	52.5%	25 – 32
3	40%	19 – 24
2	27.5%	13 – 18
1	0%	0 – 12