No graphing calculator is allowed for this problem other than for arithmetic.
It is recommended that you take no more than 30 minutes for this problem.

3. An hourglass is in the shape of a double cone as shown to the right. The bottom cone is filled with sand when it is turned upside down, sand slowly drops to the lower cone. The device is used as a timer. The red cone is a right circular cone while the blue cone is not. At height h, measured in inches from the opening in the cone, the radius r, also measured in inches, is approximate for the blue cone and exact for the red cone and are shown in the table below.

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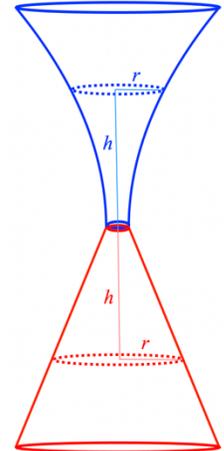
h	0	1	2	3	4
r_{blue}	0.1	0.3	1.0	2.5	4
r_{red}	0.1	1.1	2.1	3.1	4.1

(a) Write an expression that relates the linear relation of the red cone's radius to its height.

Your Score _____

(b) Use the trapezoidal rule to approximate the average radius of the blue cone.

Your Score _____

(c) Horizontal planes are draw perpendicular to the height in the red cone creating circular areas of sand. Find the average area of all these circles.

Your Score _____

(d) Sand flows through the hourglass from blue to red. When h is 1.5 inches, the radius of the sand in the blue cone is decreasing at the rate of 0.2 inches per minute. Approximate how fast the height of the sand is decreasing at that moment in time.

Your Score _____

(e) Suppose sand flows through the hourglass from red to blue. The radius of the sand in the red cone is decreasing at the rate of 0.15 inches per minute. Determine how fast the height of the sand is decreasing at that moment in time.

Your Score _____

(f) If the blue cone is completely filled with sand, use a right Riemann sum to approximate its volume. Show how you got your answer.

Your Score _____

(g) If the red cone is completely filled with sand, find its volume. Show how you got your answer.

Your Score _____

(h) The blue cone can be modeled by $r = \dfrac{h^2}{4} + \dfrac{1}{10}$. If it completely filled with sand, find its volume.

<div align="right">Your Score _____</div>

(i) Since it is an hourglass, it should take one hour to empty the blue cone. Based on answer (h), what is the rate that the cone is emptying in cubic inches per minute?

<div align="right">Your Score _____</div>

(j) Use your answer in (i) to determine how quickly the height of the sand in the blue cone is decreasing when its height is 2 inches.

<div align="right">Score _____</div>

(k) Use your answer in part (h) to write, but not solve an equation involving the height k of the sand in the red cone when it is completely filled.

<div align="right">Score _____</div>

(l) As sand flows out of the blue cone, the circumference of the sand level decreases. Using the answer in part (j) determine the rate of change of the sand's circumference when $h = 2$ inches.

Your Score _____

(m) A drop of water works its way down the red cone in the same vertical plane. Write an expression that represents the distance that it travels.

Your Score _____

Grading:

Grade yourself according to the rubric that either your teacher gave you or you can find in the MasterMathMentor.com website. Be strict with yourself. Additional grading tips are given in the accompanying YouTube video for this problem.

Section	Pts available	Your score
a	1	
b	2	
c	3	
d	3	
e	3	
f	2	
g	3	

Section	Pts available	Your score
h	3	
i	1	
j	4	
k	3	
l	3	
m	1	
Total	**32**	

There are 32 points available for this question. There is no exact formula for what number of points constitutes a 5, 4, 3, 2, or 1 on the A.P. Exam. However, these percentages are what have been used in the past based on exams released by the College Board. While you can extrapolate for just this question, realize that it tests only a limited number of AP topics. It is recommended that you do a number of questions in this series, combine your results, total your points, and then use these percentages to get a feel for how you will do in the AP exam, and more importantly, what concepts you need to strength to improve your score.

Grade	Percentage	This Question
5	70%	22 – 32
4	52.5%	17 – 21
3	40%	13 – 16
2	27.5%	9 – 12
1	0%	0 – 8