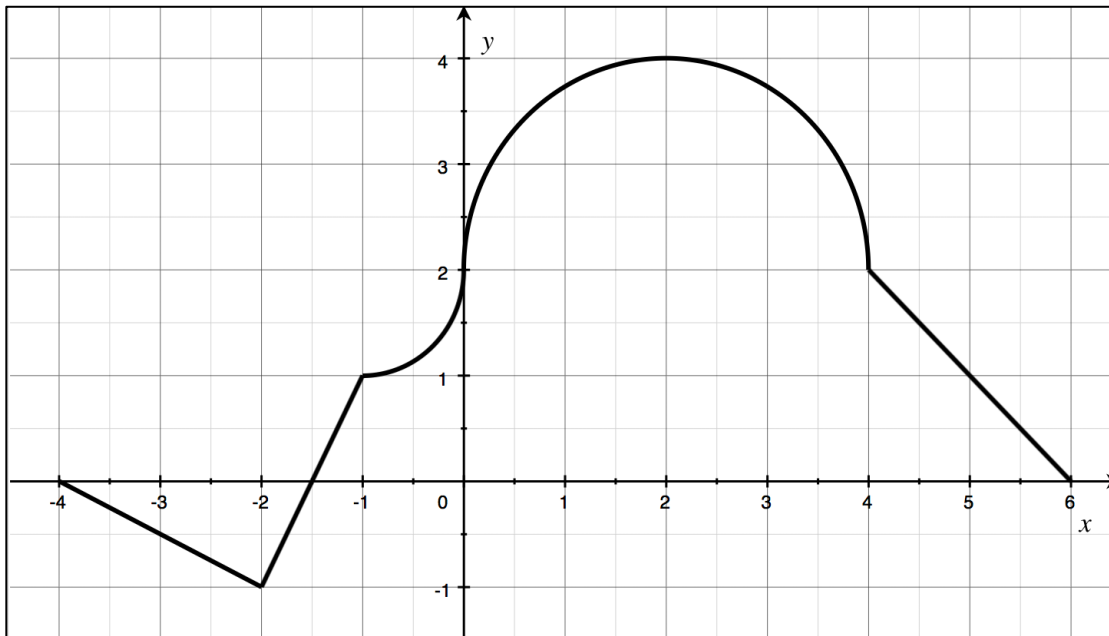




Super Free-Response Practice AB Question 4

No graphing calculator is allowed for this problem.
It is recommended that you take no more than 40 minutes for this problem.



Graph of f

4. The continuous function f is defined on the closed interval $-6 \leq x \leq 6$. The figure shows a portion of the graph of f , consisting of 3 line-segments, a semi-circle centered at the point $(2, 2)$ and a quarter-circle centered at the point $(-1, 2)$.

(a) What is the minimum number of locations where f has a horizontal tangent line? Explain.

Your Score _____

(b) What is the minimum number of locations where f has a vertical tangent line? Explain.

Your Score _____

(c) Let $g(x) = f'(x)f''(x)$ for $-4 \leq x \leq 4$. For what values of x is $g(x) > 0$? Justify your answer.

Your Score _____

(d) Find the value of $\int_0^4 f(x) dx$

Your Score _____

(e) If $\int_{-6}^0 f(x) dx = 2 + \frac{\pi}{4}$, find $\int_{-6}^{-4} f(x) dx$

Your Score _____

(f) Find $\int_{-2}^{-1} [4f'(x) - 3] dx$

Your Score _____

(g) Let $F(x)$ be defined on $[-4, 6]$ and $F(x) = F(0) + \int_0^x [f(t)] dt$. Express $F(6) - F(-4)$ as a definite integral. Show your reasoning.

Your Score _____

(h) Using your answer in (g), find the value of $F(6) - F(-4)$.

Your Score _____

(i) Find intervals where F is increasing. Justify your answer:

Your Score _____

(j) If $F(0) = 2$, find the maximum value of F .

Your Score _____

(k) For what values of x does F have an inflection point? Explain your reasoning.

Your Score _____

(l) What is the numerical difference between the average rate of change of F on $[-4, 6]$ and the average value of f on $[-4, 6]$? Explain. Explain.

Your Score _____

(m) Find the equation of the tangent line to F at $x = 2$.

Your Score _____

(n) Find $\lim_{x \rightarrow 2} \frac{F(3x) - F(2x) - x}{\tan \pi x}$

Your Score _____

(o) If applicable, apply Rolle's theorem to f on $[-4, 6]$. If it is not applicable, explain.

Your Score _____

(p) On the graph of f , the section between $x = 2$ and $x = 4$ is divided into intervals, $(2, 2.25)$, $(2.25, 2.5)$, $(2.5, 2.75)$, $(2.75, 3)$, $(3, 3.25)$, $(3.25, 3.5)$, $(3.5, 3.75)$ and $(3.75, 4)$. For which of these intervals does the conclusion of the Mean-Value Theorem hold for $F(x)$? Show the work that leads to your conclusion.

Your Score _____

Grading:

Grade yourself according to the rubric that either your teacher gave you or you can find in the MasterMathMentor.com website. Be strict with yourself. Additional grading tips are given in the accompanying YouTube video for this problem.

Section	Pts available	Your score
a	2	
b	2	
c	3	
d	2	
e	3	
f	3	
g	2	
h	2	

Section	Pts available	Your score
i	2	
j	3	
k	3	
l	2	
m	3	
n	3	
o	2	
p	3	
Total	40	

There are 40 points available for this question. There is no exact formula for what number of points constitutes a 5, 4, 3, 2, or 1 on the A.P. Exam. However, these percentages are what have been used in the past based on exams released by the College Board. While you can extrapolate for just this question, realize that it tests only a limited number of AP topics. It is recommended that you do a number of questions in this series, combine your results, total your points, and then use these percentages to get a feel for how you will do in the AP exam, and more importantly, what concepts you need to strength to improve your score.

Grade	Percentage	This Question
5	70%	28 – 40
4	52.5%	21 – 27
3	40%	16 – 20
2	27.5%	11 – 15
1	0%	0 – 10