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Super Free-Response Practice AB Question 5

No graphing calculator is allowed for this problem.

It is recommended that you take no more than 45 minutes for this problem.

5. An island country is experiencing a virus. It has developed a vaccine that is expected to counteract the virus. Let x represent the number of people vaccinated, measured in tens of thousands and y represent the number of new infections, also measured in tens of thousands.

Based on clinical trials, the relationship between x and y is expected to be the differential equation.

$$\frac{dy}{dx} = 10y - 15x.$$

- (a) On the axes provided, sketch a slope field for the differential equation at the 12 gridpoints.



Your Score _____

- (b) Find the equation of the line tangent to the curve describing the relationship between infections and vaccinations when 1,000 people are vaccinated and 2,000 people have been infected. Graph it on the slope field.

Your Score _____

(c) Find $\frac{d^2y}{dx^2}$ in terms of x and y .

Your Score _____

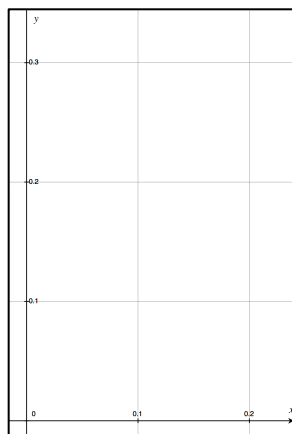
(d) Suppose at a given time, there were 3,000 people vaccinated and 4,000 current infections. Use $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ to describe the growth of the virus.

Your Score _____

(e) Suppose at a given time, there were 4,000 people vaccinated and 6,000 current infections. Does this mean that the number of infections is at a relative maximum? Justify your answer.

Your Score _____

(f) On the graph provided, shade in the area where $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} < 0$.



Your Score _____

(g) Explain the significance of the shaded region in (f) in the context of the problem.

Your Score _____

(h) Find the values of the constants m and b for which $y = mx + b$ is a solution to the differential equation.
Graph the line.

Your Score _____

(i) Explain the significance of the line in the context of the problem.

Your Score _____

(j) Suppose that the country can vaccinate 1,000 people a day. At what rate will the infections change if there are 5,000 people vaccinated and 20,000 people infected?

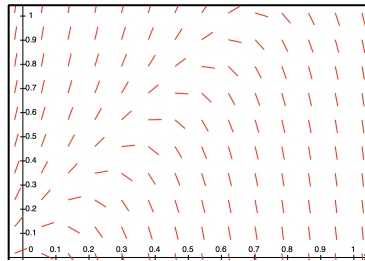
Your Score _____

The country has another vaccine with different technology that they believe will be more effective than the one previously investigated. As before, let x represent the number of people vaccinated, measured in tens of thousands and y represent the number of new infections, also measured in tens of thousands.

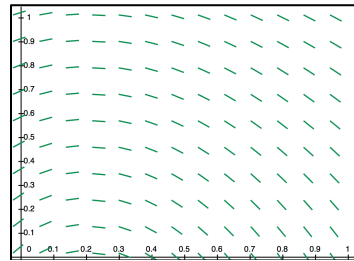
Based on clinical trials, the relationship between x and y is expected to be the differential equation

$$\frac{dy}{dx} = \frac{3x^2 - 6x + 1}{e^y}.$$

- (k) Show are the slope fields for the expected virus growth based on the older vaccine and the newer vaccine. Explain briefly why there is more confidence in the ultimate effectiveness of the newer vaccine



Older vaccine



newer vaccine

Your Score _____

- (l) At the time the vaccine is introduced, there are 10,000 people infected. If f is the particular solution of the differential equation based on that information, write an equation to the tangent line to f passing through that point.

Your Score _____

- (m) Use your answer in (l) to predict how many people will be infected when 10,000 people are vaccinated. Does this give good evidence that the vaccine is effective? Explain.

Your Score _____

(n) Use the information in (l) to solve the differential equation.

Your Score _____

(o) Use your answer in (n) to predict how many people will be infected when 10,000 people are vaccinated. Does this give good evidence that the vaccine is effective? Explain.

Your Score _____

(p) Scientists predict that fewer than 2,000 vaccines need to be given for the number of infections to reach its peak. Explain why this is true.

Your Score _____

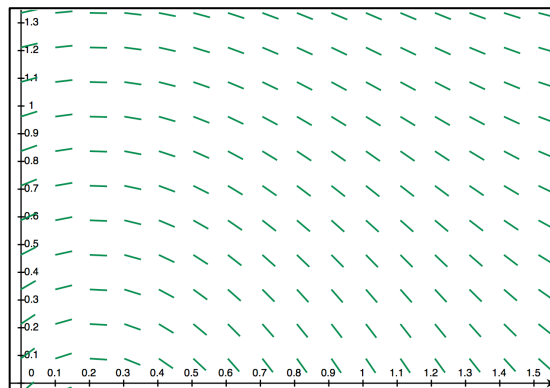
(q) Based on your answer from part (n), write, but do not solve an equation using only the variable v , the number of vaccinations that must be given for the number of new infections to be zero.

Your Score _____

- (r) Based on the answer from part (n), scientists predict that between 10,000 and 20,000 vaccinations must be given for the number of new infections to disappear. Explain mathematically why this is true.

Your Score _____

- (s) A slope field for the DEQ is given. Sketch your tangent line from part (l) and your solution equation from part (n).



Your Score _____

Grading:

Grade yourself according to the rubric that either your teacher gave you or you can find in the MasterMathMentor.com website. Be strict with yourself. Additional grading tips are given in the accompanying YouTube video for this problem.

| Section | Pts available | Your score |
|---------|---------------|------------|
| a | 3 | |
| b | 3 | |
| c | 1 | |
| d | 3 | |
| e | 3 | |
| f | 3 | |
| g | 2 | |
| h | 4 | |
| i | 1 | |
| j | 4 | |

| Section | Pts available | Your score |
|--------------|---------------|------------|
| k | 1 | |
| l | 2 | |
| m | 2 | |
| n | 3 | |
| o | 2 | |
| p | 2 | |
| q | 2 | |
| r | 3 | |
| s | 2 | |
| Total | 46 | |

There are 46 points available for this question. There is no exact formula for what number of points constitutes a 5, 4, 3, 2, or 1 on the A.P. Exam. However, these percentages are what have been used in the past based on exams released by the College Board. While you can extrapolate for just this question, realize that it tests only a limited number of AP topics. It is recommended that you do a number of questions in this series, combine your results, total your points, and then use these percentages to get a feel for how you will do in the AP exam, and more importantly, what concepts you need to strength to improve your score.

| Grade | Percentage | This Question |
|-------|------------|---------------|
| 5 | 70% | 32 – 46 |
| 4 | 52.5% | 24 – 31 |
| 3 | 40% | 18 – 23 |
| 2 | 27.5% | 13 – 17 |
| 1 | 0% | 0 – 12 |