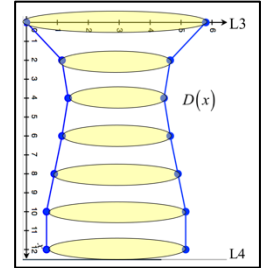




Super Free-Response Practice AB Question 6

**A graphing calculator is permitted for parts of this problem.
It is recommended that you take no more than 45 minutes for this problem.**

6. A patient is suffering from severe spinal stenosis, a narrowing of the spinal canal in the middle of the lumbar spine between the L3 and L4 vertebrae as shown on the right. A Magnetic Resonance Image (MRI) shows that the diameter of the spinal column at selected distances x in millimeters from L3 is given. It is assumed that this diameter $D(x)$ is a twice-differentiable function as given in the table below. It is also assumed that the spinal canal is in one plane.



Distance from L3 x (mm)	0	2	4	6	8	10	12
Diameter $D(x)$ (mm)	5.8	3.5	3.1	3.5	4.0	4.5	4.5

- (a) Use data from the table to find an approximation for $D'(2)$. Show the computations that lead to your answer. Indicate units of measure.

Your Score _____

- (b) Write an integral expression in terms of $D(x)$ that represents the average radius, in mm, of the spinal canal between L3 and L4.

Your Score _____

- (c) The answer to part (b) is to be approximated first with a right Riemann sum and then a midpoint Riemann sum, each with 3 subintervals of equal length. Find the difference in these calculations.

Your Score _____

(d) Explain why there must be at least one value k for $6 < k < 10$, such that $D'(k) = 0.25$.

Your Score _____

(e) Explain why there must be at least one value x for $0 < x < 12$, such that $D''(x) = 0$.

Your Score _____

(f) Write an integral expression in terms of $D(x)$ that represents the volume, in mm^3 of the spinal canal between L3 and L4.

Your Score _____

(g) The answer to part (f) is to be approximated with a left Riemann sum with 6 subintervals of equal length. Find the value of this approximation and show how you arrive at your answer.

Your Score _____

(h) If $0 \leq k \leq 12$, write an expression that represents how fast the volume of the spinal canal between L3 and L4 is changing at $x = k$.

Your Score _____

- (i) It can be determined that the diameter of the spinal column between L3 and 2 millimeters short of L4 can be modeled by the function $f(x) = 6 - 2xe^{-x/4}$ where x and $f(x)$ are measured in millimeters. Find $f'(2)$ and explain its meaning in the context of the problem situation.

Your Score _____

- (j) Using f , find the narrowest distance between the walls of the spinal canal. Justify your answer.

Your Score _____

- (k) Using f , find the value of x where the diameter of the spinal canal is growing the fastest. Justify your answer.

Your Score _____

- (l) Using f , find the volume of the spinal canal between L3 and 2 millimeters short of L4.

Your Score _____

- (m) When the spinal canal is narrowing between L3 and L4, write an equation that finds the value k such that $x = k$ divides the volume of the spinal canal into two equal volumes.

Your Score _____

-
- (n) Greater detail is needed and the MRI shows that for $10 \leq x \leq 12$, the function that models the diameter of the spinal canal has first derivative given by $D'(x) = (8x - 88)(\cos \pi x)$. Find $D(11)$.

Your Score _____

-
- (o) For $10 \leq x \leq 12$, is the answer to (n) the narrowest the spinal canal will be? If so, explain why. If not, determine the narrowest diameter of the spinal canal.

Your Score _____

Grading:

Grade yourself according to the rubric that either your teacher gave you or you can find in the MasterMathMentor.com website. Be strict with yourself. Additional grading tips are given in the accompanying YouTube video for this problem.

Section	Pts available	Your score
a	2	
b	2	
c	4	
d	2	
e	3	
f	3	
g	3	
h	2	

Section	Pts available	Your score
i	3	
j	4	
k	3	
l	2	
m	1	
n	3	
o	3	
Total	40	

There are 40 points available for this question. There is no exact formula for what number of points constitutes a 5, 4, 3, 2, or 1 on the A.P. Exam. However, these percentages are what have been used in the past based on exams released by the College Board. While you can extrapolate for just this question, realize that it tests only a limited number of AP topics. It is recommended that you do a number of questions in this series, combine your results, total your points, and then use these percentages to get a feel for how you will do in the AP exam, and more importantly, what concepts you need to strength to improve your score.

Grade	Percentage	This Question
5	70%	28 – 40
4	52.5%	22 – 37
3	40%	16 – 21
2	27.5%	11 – 15
1	0%	0 – 10