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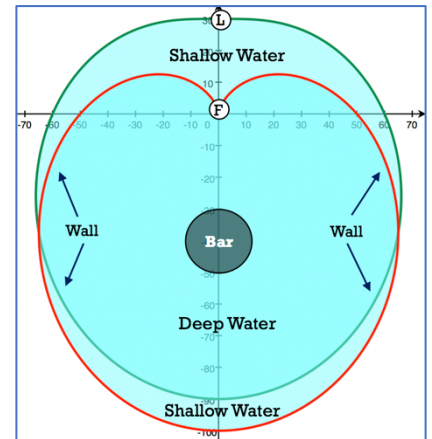
Created by Stu Schwartz

## Super Free-Response Practice BC Question 1

A graphing calculator is allowed for this problem.

It is recommended that you take no more than 45 minutes for this problem.

1. A resort has a large swimming pool. It is constructed such that are 2 sections: a large deep-water swimming area with a 20-foot diameter circular island bar in the center and 2 shallow wading areas on the outside for young children. The sections are separated by a submerged wall that people can swim or float over. There is a fountain at the point  $(0, 0)$  and a lifeguard positioned at the rectangular point  $(0, 30)$ . The border of the areas are the polar curves  $r_1 = 60 - 30\sin\theta$  and  $r_2 = 50 - 50\sin\theta$  where  $r$  is measured in feet as shown to the right.



- (a) Determine the polar points where the two curves intersect.

Your Score \_\_\_\_\_

- (b) There is a walkway that is tangent to the outside polar curve at  $\theta = \pi$ . Find the slope of this line.

Your Score \_\_\_\_\_

- (c) Find the equation of the walkway line.

Your Score \_\_\_\_\_

- (d) There are two shelves on the submerged wall above the  $x$ -axis for which adults can place drinks. If these shelves are tangent horizontally to the wall, at what values of  $\theta$  are they located? Show how you get your answers.

Your Score \_\_\_\_\_

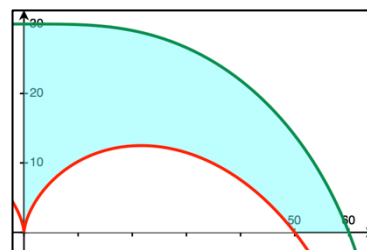
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- (e) Using correct units, find the value of  $\frac{dr_2}{d\theta}$  at  $\theta = \frac{4\pi}{3}$  and describe its significance in the context of the problem.

Your Score \_\_\_\_\_

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- (f) Jack swims along the wall separating the deep and shallow sections for  $\frac{\pi}{2} \leq \theta \leq \frac{7\pi}{6}$ . Jack swims in such a way that the distance between the fountain and himself increases at the rate of 2 feet per second. Find the rate at which the angle  $\theta$ , measured in degrees, changes with respect to time at the instance Jack is at  $\theta = \pi$ .

Your Score \_\_\_\_\_

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- (g) A section of the shallow-water pool is shown by shading in the figure to the right. Find the values of  $\theta$  that generate this section. Show how you got your answer.



Your Score \_\_\_\_\_

(h) A ray in the form  $\theta = k$  divides the region described in (g) into two equal areas. Write, but do not solve an equation involving one or more integrals that give the value of  $k$ .

Your Score \_\_\_\_\_

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(i) For each  $\theta$ , let  $S(\theta)$  be the distance between the points with polar coordinates  $(r_1, \theta)$  and  $(r_2, \theta)$ . Write an expression for  $S(\theta)$  and then find  $S_{\text{avg}}$ , the average distance between the polar points in the region described in part (g).

Your Score \_\_\_\_\_

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(j) Find the area of the deep section of the pool.

Your Score \_\_\_\_\_

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(k) Find the area of the entire pool.

Your Score \_\_\_\_\_

(l) The shallow section is 2-feet deep and the deep section is 5-feet deep. The pool is filled with 2 high pressure hoses that pump in 22.5 gallons/minute ( $3 \text{ ft}^3/\text{minute}$ ) each. How long will it take to fill the pool?

Your Score \_\_\_\_\_

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(m) If Jack swims along the wall of the deep-water section of the pool, how far does he swim?

Your Score \_\_\_\_\_

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(n) If Jill walks along the perimeter of the pool, how far does she walk?

Your Score \_\_\_\_\_

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(o) Write an expression that represents the distance from the lifeguard to any child in the upper shallow section who is against the wall of the deep -water section.

Your Score \_\_\_\_\_

(p) For  $0 \leq \theta \leq \frac{\pi}{2}$ , find the value of  $\theta$  that minimizes the distance expressed in (o). What is the minimum distance?

Your Score \_\_\_\_\_

**Grading:**

Grade yourself according to the rubric that either your teacher gave you or you can find in the [MasterMathMentor.com](http://MasterMathMentor.com) website. Be strict with yourself. Additional grading tips are given in the accompanying YouTube video for this problem.

Section	Pts available	Your score
a	3	
b	3	
c	2	
d	3	
e	2	
f	3	
g	2	
h	2	

Section	Pts available	Your score
i	3	
j	4	
k	4	
l	3	
m	4	
n	4	
o	2	
p	2	
<b>Total</b>	<b>46</b>	

There are 46 points available for this question. There is no exact formula for what number of points constitutes a 5, 4, 3, 2, or 1 on the A.P. Exam. However, these percentages are what have been used in the past based on exams released by the College Board. While you can extrapolate for just this question, realize that it tests only a limited number of AP topics. It is recommended that you do a number of questions in this series, combine your results, total your points, and then use these percentages to get a feel for how you will do in the AP exam, and more importantly, what concepts you need to strengthen to improve your score.

Grade	Percentage	This Question
5	70%	32 – 46
4	52.5%	24 – 32
3	40%	18 – 23
2	27.5%	13 – 17
1	0%	0 – 12