



# MasterMathMentor.com

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## Super Free-Response Practice BC Question 2

**A graphing calculator is allowed for several portions of this problem.  
It is recommended that you take no more than 35 minutes for this problem.**

2. For  $t \geq 0$ , a particle is moving along a curve so that its position at time  $t$  is  $(x(t), y(t))$ . At time  $t = 0$ , the particle is at position  $(-3, 4)$ . It is known that  $\frac{dx}{dt} = \frac{2-2t}{t^2+6t+8}$  and  $\frac{dy}{dt} = \frac{t+1}{e^t}$  for  $0 \leq t < \infty$ .

(a) Is the horizontal movement of the particle initially left or right? Explain your answer.

Your Score \_\_\_\_\_

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(b) How fast is the vertical position of the particle initially changing and accelerating?

Your Score \_\_\_\_\_

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(c) Find the slope of the path of the particle at  $t = 0$ .

Your Score \_\_\_\_\_

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(d) Find the equation of the tangent line to the path of the particle at  $t = 0$ .

Your Score \_\_\_\_\_

(e) Find the initial speed of the particle. Show how you arrived at your answer.

Your Score \_\_\_\_\_

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(f) Find the  $x$ -coordinate of the particle at  $t = 2$ . The answer must be exact (no calculator).

Your Score \_\_\_\_\_

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(g) Find the  $y$ -coordinate of the particle at  $t = 2$ . The answer must be exact (no calculator).

Your Score \_\_\_\_\_

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(h) Using the answers to (f) and (g), find the angle between the path of the particle and the horizontal at  $t = 2$ .

Your Score \_\_\_\_\_

(i) Is the particle slowing down or speeding up in the  $x$ -direction at  $t = 2$ ? Justify your answer.

Your Score \_\_\_\_\_

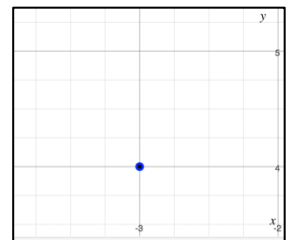
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(j) How far does the particle travel between  $t = 0$  and  $t = 2$ ?

Your Score \_\_\_\_\_

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(k) On the graph, the initial point  $(-3, 4)$  is shown. Draw the velocity and acceleration vector of the particle at  $t = 0$ .



Your Score \_\_\_\_\_

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(l) Determine the value of  $t$  when the line tangent to the path of the particle is vertical. Is the direction of the motion of the particle up or down at that time? Give a reason for your answer. **(2)**

Your Score \_\_\_\_\_

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(m) Give an argument why the parametric curve will be asymptotic to a horizontal line.

Your Score \_\_\_\_\_

(n) Determine whether the path of the particle is concave up or concave down at  $t = 0$ .

Your Score \_\_\_\_\_

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**Grading:**

Grade yourself according to the rubric that either your teacher gave you or you can find in the [MasterMathMentor.com](http://MasterMathMentor.com) website. Be strict with yourself. Additional grading tips are given in the accompanying YouTube video for this problem.

Section	Pts available	Your score
a	2	
b	2	
c	1	
d	1	
e	2	
f	4	
g	4	

Section	Pts available	Your score
h	2	
i	3	
j	2	
k	4	
l	2	
m	4	
n	3	
<b>Total</b>	<b>36</b>	

There are 36 points available for this question. There is no exact formula for what number of points constitutes a 5, 4, 3, 2, or 1 on the A.P. Exam. However, these percentages are what have been used in the past based on exams released by the College Board. While you can extrapolate for just this question, realize that it tests only a limited number of AP topics. It is recommended that you do a number of questions in this series, combine your results, total your points, and then use these percentages to get a feel for how you will do in the AP exam, and more importantly, what concepts you need to strength to improve your score.

Grade	Percentage	This Question
5	70%	25 – 36
4	52.5%	19 – 24
3	40%	14 – 18
2	27.5%	10 – 13
1	0%	0 – 9