



Super Free-Response Practice BC Question 3

No graphing calculator is allowed for this problem.

It is recommended that you take no more than 40 minutes for this problem.

3. According to the American Cancer Society, the size of a tumor is based on its length at its widest point.

T1: 2 cm or less

T2: more than 2 cm but less than 5 cm

T3: 5 cm or greater

Dr. Fanucci, an oncologist (a doctor who treats cancer) assesses the tumor's features and based on that assessment and numerous factors of the patient, he recommends a treatment plan, and tries to determine how it will grow or shrink over time. There are many factors involved but he classifies them based on the following differential equation where W is the tumor's width measured in cm, t is time measured in months, and k is a constant.

Type A: $\frac{dW}{dt} = kW$

Type B: $\frac{dW}{dt} = k(M - W)$

Type C: $\frac{dW}{dt} = k(M - W)^n$

Type D: $\frac{dW}{dt} = k(W - m)(M - W)$

(Note: although the size classification is accurate, these differential equations representing the type of growth have been made up for the purpose of this problem.)

- (a) José is told that his T3 tumor is changing proportional to current size with the constant of proportionality $k = -0.2$. If $W(0) = 6$, determine the tumor type and solve the differential equation that describes the size of his tumor. Write a statement that Jose can understand which describes what he can expect about the tumor's size in the long run. Explain mathematically why this is true.

Your Score _____

- (b) José would like to know how long Dr. Fanucci believes it will take before the tumor is less than 2 cm in size. Determine this mathematically.

Your Score _____

- (c) Alison has had her T3 tumor assessed as a type B with $M = 4$, $k = 0.5$, and $W = 10$ at $t = 0$. Write an equation for the line tangent to the graph of W at $t = 0$. Use this equation to approximate the size of the tumor at $t = 2$ months.

Your Score _____

- (d) Use $\frac{d^2W}{dt^2}$ to determine whether your answer in part (a) is an under-approximation or over-approximation of the tumor's predicted size.

Your Score _____

- (e) Donald has had his T2 tumor assessed as a type B with $M = 4$, $k = 0.5$, and $W = 2$ at $t = 0$. Solve the differential equation that describes the size of this tumor.

Your Score _____

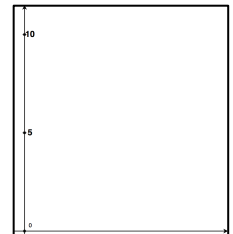
- (f) If you were Dr. Fanucci, what would you tell Donald is the long-range prognosis about the growth and of size of the tumor. Explain how you reach that conclusion

Your Score _____

- (g) Mollie has a type C T3 cancer with $k = -1$, $M = 5$, and $n = 2/3$. Based on $\frac{dW}{dt}$, what will be the doctor's preliminary prognosis about the size and spread of the tumor?

Your Score _____

- (h) Find $\frac{d^2W}{dt^2}$ and based on your answer (g), make a sketch of the general shape of the graph of W on the supplied axes. What should the doctor say to Mollie?

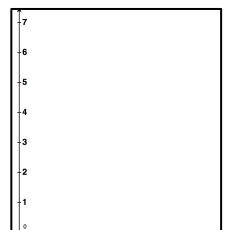


Your Score _____

- (i) Angie had her T1 tumor assessed in another hospital, but her doctor did not use the same assessment system that Dr. Fanucci uses. When she transferred to Dr. Fanucci, he took the differential equation that was given to her in the other hospital, $\frac{dW}{dt} = 0.5W \left(1 - \frac{W}{5} \right)$, and determined that it was a Type D tumor. Show his analysis and name the type of growth/decay this tumor should show.

Your Score _____

- (j) Assuming that the tumor in part (i) acts according its classification, describe to Angie how the tumor will change over time in a sentence or two. On the axes provided, draw a simple sketch of the projected size of the tumor over time.



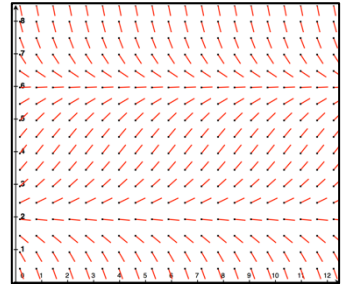
Your Score _____

(k) How large will the tumor described in part (i) be when it reaches its maximum growth rate? Show how you arrive at your answer.

Your Score _____

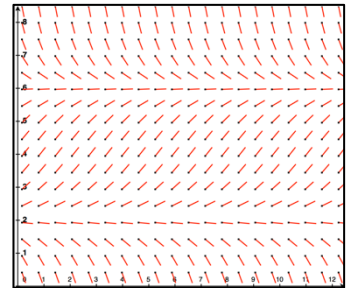
On a particular day, several patients were diagnosed with Type D tumors with varying sizes. Its parameters are: $k = 0.25$, $m = 2$, $M = 6$. To the right is a slope field of the differential equation describing the tumor's growth. For each subpart, you are given the tumor's size when diagnosed ($t = 0$). Draw the particular solution onto the given slope field, give a mathematical explanation for your answer, and generate a short prognosis for the patient, paying attention to the projected long-term size of the tumor and growth.

(l) Jason: $W = 1.5$ cm.



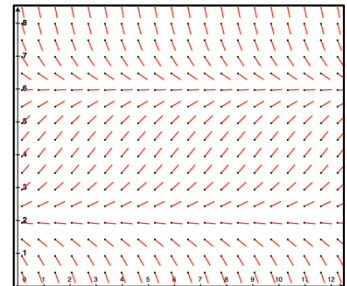
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(m) Derek: $W = 2$ cm.



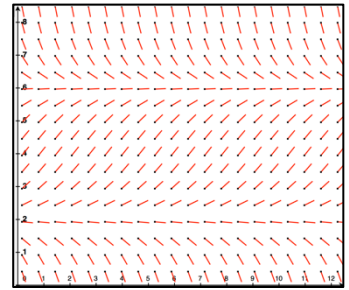
Your Score _____

(n) Helen: $W = 4.5$ cm.



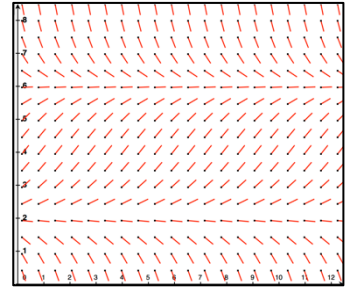
Your Score _____

(o) Helen: $W = 4.5$ cm.



Your Score _____

(p) Laura. $W = 8$ cm.



Your Score _____

(q) Laura has had some calculus so she decides to approximate the tumor's size after 3 months using Euler's Method with an increment of one month. Show the procedure and explain why this size increment with Euler's Method can lead to a false conclusion.

Your Score _____

Grading:

Grade yourself according to the rubric that either your teacher gave you or you can find in the MasterMathMentor.com website. Be strict with yourself. Additional grading tips are given in the accompanying YouTube video for this problem.

Section	Pts available	Your score
a	5	
b	2	
c	3	
d	2	
e	3	
f	2	
g	1	
h	3	
i	2	

Section	Pts available	Your score
j	3	
k	2	
l	3	
m	3	
n	3	
o	4	
p	3	
q	4	
Total	46	

There are 46 points available for this question. There is no exact formula for what number of points constitutes a 5, 4, 3, 2, or 1 on the A.P. Exam. However, these percentages are what have been used in the past based on exams released by the College Board. While you can extrapolate for just this question, realize that it tests only a limited number of AP topics. It is recommended that you do a number of questions in this series, combine your results, total your points, and then use these percentages to get a feel for how you will do in the AP exam, and more importantly, what concepts you need to strength to improve your score.

Grade	Percentage	This Question
5	70%	32 – 46
4	52.5%	24 – 32
3	40%	18 – 23
2	27.5%	13 – 17
1	0%	0 – 12