



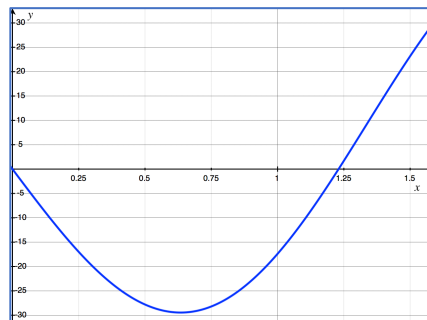
# MasterMathMentor.com

Created by Stu Schwartz

## Super Free-Response Practice BC Question 4

No graphing calculator is allowed for this problem other than for arithmetic.  
It is recommended that you take no more than 30 minutes for this problem.

4. This problem is concerned with Taylor and Power Series, and Taylor polynomials. Such problems either give a function and ask for information about its Taylor series or give a summation and ask questions about its convergence. This question will do both.



Graph of  $f^{(4)}(x)$

I. Let  $f(x) = \sin\left(\frac{3x}{2}\right) - x\cos(2x)$ . The graph of  $f^{(4)}(x)$  is shown above.

- (a) Write the first three nonzero terms and general term for the power series for  $\sin x$  about  $x = 0$ .

Your Score \_\_\_\_\_

---

(b) Write the first three nonzero terms for and general term for the power series for  $\sin\left(\frac{3x}{2}\right)$  about  $x = 0$ .

Your Score \_\_\_\_\_

(c) Write the first three nonzero terms and general term for the power series for  $\cos x$  about  $x = 0$ .

Your Score \_\_\_\_\_

---

(d) Write the first three nonzero terms for and general term for the power series for  $\cos(2x)$  about  $x = 0$ .

Your Score \_\_\_\_\_

---

(e) Write the first three nonzero terms for the power series for  $f$  about  $x = 0$ .

Your Score \_\_\_\_\_

---

(f) Find the value of  $\sum_{n=1}^5 f^{(n)}(0)$  (either fraction or decimal value).

Your Score \_\_\_\_\_

(g) Let  $P_3(x)$  be the third-degree Taylor polynomial for  $f$  about  $x = 0$ . Explain the meaning of

$$\left| P_3\left(\frac{3}{4}\right) - f\left(\frac{3}{4}\right) \right| \text{ and given the nature of the series, classify it with a name.}$$

Your Score \_\_\_\_\_

---

(h) Use information from the graph of  $y = f^{(4)}(x)$  shown above to prove that  $\left| P_3\left(\frac{3}{4}\right) - f\left(\frac{3}{4}\right) \right| < \frac{1}{2}$ .

Your Score \_\_\_\_\_

---

(i) Let  $g(x) = \sin\left(\frac{3x}{2}\right) + x \cos(2x)$ . It can be shown that this series is alternating. Let  $Q_3(x)$  be the 3<sup>rd</sup> degree Taylor polynomial for  $g$  about  $x = 0$ . Determine if the error in using  $Q_3(x)$  to approximate  $g\left(\frac{1}{2}\right)$  is less than  $\frac{1}{10}$ .

Your Score \_\_\_\_\_

---

(j) Determine the largest possible value of  $c$ ,  $c > 0$  such that the error in using  $Q_3(x)$  in part (i) to approximate  $g(c) < \frac{1}{10}$  is less than  $\frac{1}{10}$ .

Score \_\_\_\_\_

- (k) Let  $h$  be the function defined by  $h(x) = \int_0^x g(t) dt$ . Use the Taylor polynomial found in part (i) to find an approximation for the average value of  $h$  on  $\left[0, \frac{1}{2}\right]$ .

Score \_\_\_\_\_

---

II. The Taylor Series for a function  $f$  centered at  $x = 1$  is given by  $1 - \frac{2}{5}(x-1) + \frac{3}{25}(x-1)^2 - \frac{4}{125}(x-1)^3 + \dots$

- (l) Express  $f$  using Sigma notation.

Your Score \_\_\_\_\_

---

- (m) Suppose the series converges to  $f(x)$  for  $|x| < R$ , where  $R$  is the radius of convergence of the Taylor series. Use an appropriate test to find  $R$ .

Your Score \_\_\_\_\_

---

- (n) Find the interval of convergence of  $f$ . Show the steps that lead to your answer.

Your Score \_\_\_\_\_

(o) Explain why  $f(0)$  is or is not a geometric series. Use your result to either find the value of  $f(0)$  or find a lower limit for  $f(0)$ .

Your Score \_\_\_\_\_

---

(p) Find  $f^{(20)}(1)$

Your Score \_\_\_\_\_

---

(q) Write the first 3 non-zero terms of the Maclaurin series for  $e^{x-1}$ .

Your Score \_\_\_\_\_

---

(r) Use the result in (q) to write the second-degree Taylor Polynomial for  $g(x) = e^{x-1}f(x)$  about  $x = 1$ .

Your Score \_\_\_\_\_

## Grading:

Grade yourself according to the rubric that either your teacher gave you or you can find in the [MasterMathMentor.com](http://MasterMathMentor.com) website. Be strict with yourself. Additional grading tips are given in the accompanying YouTube video for this problem.

| Section | Pts available | Your score |
|---------|---------------|------------|
| a       | 2             |            |
| b       | 2             |            |
| c       | 2             |            |
| d       | 2             |            |
| e       | 3             |            |
| f       | 3             |            |
| g       | 2             |            |
| h       | 2             |            |
| i       | 2             |            |

| Section      | Pts available | Your score |
|--------------|---------------|------------|
| j            | 2             |            |
| k            | 3             |            |
| l            | 3             |            |
| m            | 3             |            |
| n            | 3             |            |
| o            | 3             |            |
| p            | 3             |            |
| q            | 2             |            |
| r            | 3             |            |
| <b>Total</b> | <b>45</b>     |            |

There are 45 points available for this question. There is no exact formula for what number of points constitutes a 5, 4, 3, 2, or 1 on the A.P. Exam. However, these percentages are what have been used in the past based on exams released by the College Board. While you can extrapolate for just this question, realize that it tests only a limited number of AP topics. It is recommended that you do a number of questions in this series, combine your results, total your points, and then use these percentages to get a feel for how you will do in the AP exam, and more importantly, what concepts you need to strengthen to improve your score.

| Grade | Percentage | This Question |
|-------|------------|---------------|
| 5     | 70%        | 32 – 45       |
| 4     | 52.5%      | 24 – 31       |
| 3     | 40%        | 18 – 23       |
| 2     | 27.5%      | 12 – 17       |
| 1     | 0%         | 0 – 11        |