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*calculator allowed

A wake-up call on the dangers of closing our eyes to sleep deprivation

He had one hour's sleep the night before. Only one. One hour's sleep before stepping into the cockpit of a passenger jet as captain of the crew flying it from South America to Europe. He said so himself, on the black box recording retrieved long after the plane went down in the Atlantic, killing three Irish doctors and 225 other passengers. The crash happened in 2009, on an overnight flight to Paris from Rio de Janeiro. "I didn't sleep enough last night," the captain, a 58-year-old named Marc Debois, is heard to say on the recording before the disaster happened. "One hour. It's not enough." If he had been the only sleep-deprived crew member in the cockpit, that would have been bad enough. But, apparently, all of the pilots theoretically in charge that night had under-slept, due (according to reports) to having been out in the town in Rio with their wives and girlfriends the previous night. When things began to go wrong on the flight, all cockpit crew demonstrated the effects of lack of sleep. First of all, the captain, who was on a break when the plane's electronics began to yell a stall warning, took more than a minute to respond to calls from his colleagues to take over and solve the problem. When he eventually did take over, the other crew members couldn't explain to him, clearly and succinctly, what had happened, because they, too, were groggy. The end result was that the crew took precisely the wrong action, pulling the plane's nose up, failing to realize that this was exacerbating the problem even as the Airbus A330 headed for the ocean. In the three and a half minutes before the jet hit the water, they failed to get on top of the issue enough to rescue themselves and their passengers. People died, not because the pilots were drunk, but because they were tired.

The problem below revolves around the proper amount of sleep people need.

The recommended amount of nightly sleep in hours for a person is given by the formula $S(a) = 14.7a^{-0.146}$ where a is their age in years. The formula is valid for $2 \leq a \leq 65$.

*a) Find $S'(18)$. Using correct units, explain the meaning of $S'(18)$ in the context of the problem situation.

*b) Find the average rate of change of S over the interval $2 \leq a \leq 65$. Find the age when the instantaneous rate of change of S is equal to the average rate of change of S over the interval $2 \leq a \leq 65$. Show how you arrive at your answer.

*c) A. Jing, who is 50 years old, has a condition called rapid aging. His body is aging at 1.5 times its normal rate. At that time, what is the rate of change of amount of nightly sleep he needs?

*d) Find the average amount of nightly sleep a person should get between ages 25 and 60.

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*a) Find $S'(18)$. Using correct units, explain the meaning of $S'(18)$ in the context of the problem situation.

$$S'(a)=(14.7)(-0.146)a^{-1.146} \implies S'(18)=(14.7)(-0.146)(18^{-1.146})=-0.078.$$
When a person is 18 years old, the amount of sleep he needs nightly
is decreasing by 0.078 hours a year.

*b) Find the average rate of change of S over the interval $2 \le a \le 65$. Find the age when the instantaneous rate of change of S is equal to the average rate of change of S over the interval $2 \le a \le 65$. Show how you arrive at your answer.

$$\text{Average rate of change: } \frac{S(65)-S(2)}{65-2}=\frac{7.992-13.285}{63}=-0.084$$
$$S'(a)=(14.7)(-0.146)a^{-1.146}=-0.084$$
$$a^{-1.146}=\frac{-0.084}{(14.7)(-0.146)} \implies a=e^{\frac{\ln(0.039)}{-1.146}} \approx 16.9 \text{ years old}$$

*c) A. Jing, who is 50 years old, has a condition called rapid aging. His body is aging at 1.5 times its normal rate. At that time, what is the rate of change of amount of nightly sleep he needs?

$$\left[\frac{dS}{dt}\right]_{t=50}=\left(\frac{dS}{da}\cdot\frac{da}{dt}\right)\Big|_{t=50}=S'(50)(1.5)=-0.036\frac{\text{hours}}{\text{year}}$$

*d) Find the average amount of nightly sleep a person should get between ages 25 and 60.

$$\frac{\int_{25}^{60} S(t)}{60-25}=\frac{299.101}{35}=8.546 \text{ hours}$$